Misallocation with Capital Heterogeneity *

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December, 2024

Abstract

We argue that accounting for capital heterogeneity is important for understanding the sources and costs of misallocation. We prove that, conditional on the same observables, further disaggregation of capital types will always lead to a higher measured cost of misallocation. Quantitatively, accounting for capital heterogeneity increases measured costs of misallocation by 7 p.p. (19%) in the U.S. and 6 p.p. (24%) in India. Across countries, structures are consistently more misallocated than equipment. We then estimate a dynamic model to disentangle sources of misallocation that can explain the additional measured misallocation and the efficiency differences between equipment and structures. Results indicate that adjustment costs and imperfect information cannot fully explain the additional misallocation or why structures are more misallocated. Heterogeneous financial constraints and tax policies may contribute to the higher misallocation of structures, while heterogeneous technology and measurement errors play only modest roles.

*We are indebted to Ryan Chahrour, Will Cong, Ezra Oberfield, Kristoffer Nimark and Mathieu Taschereau-Dumouchel for their continuous guidance and support. We also thank Justin Bloesch, Johannes Boehm, Julieta Caunedo, Vasco Carvalho, Joseph Kaboski, Philipp Kircher, Nathan Zorzi, University of Manchester, University of Edinburgh, University of Wisconsin-Madison (Finance), University of Wisconsin-Madison (AAE) and many seminar participants for their insights. Additionally, we thank Johannes Boehm, Nancy Chau, Ezra Oberfield, Cian Ruane, and Vidhya Soundararajan for their help in accessing and processing the India ASI data. All mistakes are our own.

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1 Introduction

Distortions in the allocation of capital across firms can reduce aggregate productivity. However, measuring these distortions is hard, and different strategies can lead to very different results. Much of the literature, including the seminal work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), relies on a convenient but probably significant assumption: different types of capital–whether machinery, buildings, or vehicles–are homogeneous. In other words, these different types of capital are perfect substitutes in a firm's production.

The issue of proper capital aggregation has been unresolved since the Cambridge-Cambridge controversies (see Cohen & Harcourt, 2003; Nunes-Pereira & Graça Moura, 2024, for reviews), after which an aggregate capital index has been commonly used in macroeconomics. However, much empirical evidence, especially from the literature on aggregate productivity and capital composition (Caselli & Wilson, 2004; Wilson, 2009), as well as estimations of housing production (Combes, Duranton, & Gobillon, 2021; Epple, Gordon, & Sieg, 2010, and others), suggests that the elasticity of substitution among capital types is distinctive finite. Hence, a question naturally arises: how does accounting for imperfect capital substitutability change our understanding of the sources and the costs of capital misallocation?[1]

In this paper, we first argue that treating all capital as homogeneous generally underestimates the extent of capital misallocation (i.e. marginal revenue product of capital dispersion). Moreover, understanding the asset-specific sources of misallocation is essential for explaining the efficiency differences across different types of capital.[2] While the implications of capital heterogeneity have been investigated in production networks (vom Lehn & Winberry, 2022), ICAPM (Gonçalves, Xue, & Zhang, 2020) and stock market (Luo, 2022), the capital misallocation literature has been silent to it.

To relax the assumption of capital homogeneity, we propose a novel static measurement framework that allows firms to use multiple types of capital in production. In this framework, different types of capital are aggregated by a constant elasticity of substitution (CES) aggregator. When elasticity of capital substitution approaches infinity, our framework nests the model in Hsieh and Klenow (2009), where the CES capital bundle collapses into an aggregate capital index as the sum of all types of capital.

The main theoretical insight from the measurement framework is as follows: the measured costs of capital misallocation decrease as the elasticity of capital substitution increases. The underlying intuition is that, assuming a larger elasticity of capital substitution, economists believe firms can create a larger CES capital bundle with a fixed set of capital inputs (since, intuitively, different types of capital are easier to substitute). As a result,

[1] In this paper, we use imperfect capital substitutability, capital heterogeneity, heterogeneous capital interchangebly. We also use perfect substitutability, capital homogeneity and homogeneous capital interchangebly.

[2] The frictions studied in the misallocation literature are typically assumed to affect all types of capital equally, and thus cannot explain the empirical finding that the dispersion of marginal products varies significantly across different capital types.

measured firm-level productivity declines, which eventually leads economists to consider a smaller counterfactual efficient output. As the observed realized aggregate output moves closer to the counterfactual efficient output with a larger elasticity of capital substitution, economists will conclude that the costs of capital misallocation are smaller. This theoretical insight suggests that whenever the elasticity of capital substitution is finite, measuring capital misallocation at a more disaggregated level uncovers "within-firm", capital-specific distortions.[3]

To apply the framework to data, the first step is to estimate the value of the elasticity of capital substitution. This parameter has been of interests since the seminal work of Sato (1967), after which a small literature has tried to identify the elasticity with aggregate time series or sector-level panel data. Given the data limitation, the best we can do is to estimate the firm-level elasticity of equipment and structures substitution in the US. In our main specification, we regress the ratio of sector-level equipment' and structures' user costs on the firm-level ratio of equipment and structures quantities. To disentangle the true elasticity from the bias of technical change (León-Ledesma, McAdam, & Willman, 2010), we implement exogenous variation in capital prices via a "shift-share" approach. The estimated results are around 0.3 across different specification, suggesting that different forms of capitals are strong complements.[4]

Applying the estimated elasticity of capital substitution and our framework to data, we focus on three datasets: Compustat North America (US), India Annual Survey of Industries (India), and Orbis Global Financials for Industrial Companies. By disaggregating the total fixed assets into equipment and structures, our measurement framework measures around 7 percentage points (19%) more misallocation in the US and 6 percentage points more (24%) in India. Further disaggregate capital into six different types of assets in ASI, the difference between the two measures can range as high as 40 percentage points, equivalently, 90% of the total misallocation. Results from the Orbis data show that this difference also holds in Australia, China, Canada, France and Japan ranging from 3% to 26%.[5]

To further explore the allocative efficiency differences between different types of capital, we apply the methodology in D. R. Baqaee and Farhi (2020) to decompose aggregate misallocation by capital type. We found that although the marginal revenue product of equipment is three times less dispersed than that of structures, equipment contributes more to aggregate misallocation due to its larger share in firms' production—almost twice

[3] "Within-firm" holds true when we ignore the capital bundle aggregation heterogeneity. In fact, we should also consider that different composition of different types of capital will lead to heterogeneous capital after the CES aggregation. Hence, using only the total fixed asset to measure the capital misallocation not only miss the capital imperfect substitutability, but also ignore this type of capital heterogeneity.

[4] There are surprisingly few papers that estimate firm-level elasticity of capital substitution from the literature. The first existing estimate can be attributed to Boddy and Gort (1971), who use variation in capital prices and capital expenditure to identify the elasticity of substitution between equipment and structure at the sector-level to be 1.72. At the occupation level, using 24 capital goods, over 9 occupations, Caunedo, Jaume, and Keller (2023a) estimated that $\gamma = 1.13$. Moreover, Ahlfeldt and McMillen (2014) estimate for elasticity between land and capital in housing production and argue that land and capital are closed to a Cobb-Douglas form.

[5] We argue that our estimation is likely conservative, as in reality, total fixed asset can be further disaggregated into more than just six types, e.g. Wilson (2009) who uses more than 19 different assets in estimation.

that of structures.

The empirical evidence of heterogeneous allocative efficiency between equipment and structures suggests the need to consider capital-specific frictions or distortions when studying sources of misallocation. The literature typically assumes that frictions are common to all types of capital, which cannot explain our decomposition results (e.g., markups in Peters, 2020).[6] Moreover, we also want to know what frictions/distortions contribute to the additional misallocation measured with finite elasticity of capital substitution.

We extend our static measurement framework to incorporate dynamic investments in equipment and structures with frictions. Specifically, we explicitly model quadratic equipment and structure adjustment costs (Asker, Collard-Wexler, & De Loecker, 2014) and imperfect information, where firms learn about future productivity from a noisy signal as Bayesian learners (David, Hopenhayn, & Venkateswaran, 2016; Ropele, Gorodnichenko, & Coibion, 2023). In addition to these two frictions, we introduce HK09-type residual distortions for equipment and structures to capture the remaining misallocation.

For model estimation, we extend the methodology of David and Venkateswaran (2019) by using the Simulated Method of Moments (SMM) to estimate our dynamic model with moments from Compustat. While we consistently calibrate most parameters, we estimate the model using three values for the elasticity of substitution between equipment and structures: 0.3, 1, and 4. This allows us to assess which frictions contribute more to misallocation when elasticity is lower. With the estimated model, we assess the counterfactual impact of each friction on aggregate misallocation by isolating their individual effects.

Results from the counterfactual analysis indicate that adjustment costs and information frictions cannot account for the additional misallocation observed when elasticity decreases. Together, equipment and structural adjustment costs contribute approximately 2% of aggregate TFP loss, regardless of how elasticity is calibrated. Similarly, information frictions consistently lead to a 1% TFP loss. Furthermore, adjustment costs explain only a small fraction of the efficiency differences between structures and equipment: structural adjustment costs alone result in greater MRPS dispersion than the MRPE dispersion driven by equipment adjustment costs, although this difference is modest compared to what is observed in the data. Imperfect information also fails to explain the efficiency differences, as it generates nearly identical MRPS and MRPE dispersions.

Instead, the two residual distortions emerge as the primary factors explaining the additional misallocation and the greater dispersion of MRPS compared to MRPE. Consequently, we investigate the economic forces within these residual distortions that account for these observations. We primarily consider four candidates: heterogeneous financial frictions, tax policies, heterogeneous technology, and potential measurement errors.

First of all, to incorporate heterogeneous financial frictions into our model, we assume that firms need costly liquidity in order to operate. Unlike the standard modeling of the

[6] Understanding sources of misallocation for each type of capital is important, since a common friction acting on all types of capital can not explain different dispersion of marginal products of different types of capital. If a common friction drives MRPE and MRPS dispersion simultaneously and identical, counterfactual implication that the covariance of the marginal revenue product of equipment and structures be exactly one.

financial friction in the literature (Buera, Kaboski, & Shin, 2011; Gopinath, Kalemli-Özcan, Karabarbounis, & Villegas-Sanchez, 2017; Midrigan & Xu, 2014; Moll, 2014), we adopt a stylized continuous liquidity cost that is jointly determined by equipment, structures, and the firm's leverage ratio. Additionally, we assume that firms with a higher leverage ratio face greater liquidity costs. The optimality condition for liquidity asset holding suggests a structural regression, where we regress equipment and structures on the leverage ratio. When applying this approach to Compustat data, our results indicate that firms with a higher leverage ratio tend to hold more structures. This finding aligns with the intuition presented by Sraer and Thesmar (2023) and Kermani and Ma (2023), which posits that structures are more frequently used as collateral.

Second, following House and Shapiro (2008) and Zwick and Mahon (2017), we study the effect of the tax policy known as "bonus" depreciation on the dispersions of MRPE and MRPS. The "bonus" depreciation accelerates the schedule for when firms can deduct the cost of investment purchases from their taxable income. However, this tax policy is not applicable to most structures investments. Using this tax policy, we find that a one standard deviation increase in the policy is associated with a 0.3 standard deviation decrease in MRPE dispersion, while it has no statistically significant effect on MRPS dispersion.

Technology heterogeneity and measurement errors can be considered part of reduced-form distortions. To quantify the contribution of heterogeneous equipment and structures elasticity, we employ three alternative methods: (i) allowing firms to use different types of capital within a sector; (ii) utilizing the GNR estimator (Gandhi, Navarro, & Rivers, 2020) for estimating firm-level input elasticity; and (iii) assuming perfect comovement between equipment, structures, and labor distortions. The first method examines the extensive margin, while the latter two methods examine the intensive margin of technology heterogeneity. Our results indicate that technology heterogeneity is not a major factor in capital misallocation. To assess measurement errors, we follow Bils, Klenow, and Ruane (2021) and find no additional errors when using equipment and structures data.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework for our analysis. Section 3 provides an overview of the three micro-level datasets used and how we infer misallocation from them. In Section 4, we estimate the elasticity of capital substitution. Section 5 presents empirical results from applying our static measurement framework to various databases. Section 6 conducts a quantitative exercise to estimate sources of asset-specific misallocation. Sections 6 and 7 detail the dynamic structural model and its estimation results. Section 8 discusses additional findings and robustness checks, while Section 9 concludes the paper.

2 Measuring Misallocation with Capital Heterogeneity

To measure misallocation with capital heterogeneity, we develop a novel static framework where different types of capital are imperfect substitutes. We begin with a baseline model of a one-sector economy and demonstrate two main theoretical results: (i) assuming perfect substitutability among different types of capital invariably leads to an underestimation of capital misallocation levels; and (ii) increasing the granularity of capital disaggregation results in a higher measured level of misallocation. We then extend our framework and theoretical findings to a multi-sector economy.

2.1 Baseline Model: One-sector Economy

We model the demand side of our framework closely following HK09, which is also the norm of this literature. Consider a static, one-sector economy with monopolistic competitive firms. The final output Y is aggregated from a set of differentiated firm-level outputs by a CES production function

$$Y = \left(\sum_{i=1}^{N} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.1}$$

where N and Y_i represent the total number of firms and their outputs. The elasticity of substitution of goods, σ is larger than one. [7]

Our modeling innovation lies in the firm's production technology. Rather than assuming a single capital K to represent a firm's total fixed assets, we allow for M types of heterogeneous capital K_{mi} and labor L_i in firm's production function

$$Y_i = A_i \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha}, \tag{2.2}$$

where A_i denotes the firm's total factor productivity, α_m indicates the intensity of usage for each type of capital, and α represents the overall capital share.

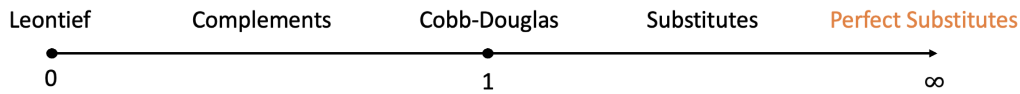
More importantly, The elasticity of capital substitution, denoted as γ, determines how easily one type of capital can be substituted for another in the production process.[8] Different values of γ indicate different production function, as it is shown in Figure 1. When $\gamma < 1$, different types of capital are gross complements, whereas $\gamma > 1$ indicates that they are gross substitutes. Three limiting cases are noteworthy. First, as $\gamma \to 0$, the capital bundle becomes Leontief, meaning different capitals must be used in fixed proportions. Second, when $\gamma \to 1$, the capital bundle reduces to what a Cobb-Douglas structure would imply.

Thirdly and more importantly, as $\gamma \to \infty$, which is commonly assumed in the literature, different types of capital become perfectly substitutable. This means that firms can replace one type of capital with another without any diminishing returns. In this limit, the production function collapses into the case in HK09 and most of the other work in the

[7] The profit maximization of the final output producer yields the inverse demand for firm-level output $Y_i = Y\left(\frac{P}{P_i}\right)^{\sigma}$ where $P = \left(\sum_{i=1}^{N} P_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the aggregate price index. These settings are standard in the capital misallocation literature.

[8] For example, in equilibrium, it measures the percentage change in the ratio of employed capital types due to a one percent change in the price ratio of these inputs.

Figure 1: Different Values of γ Determines Production Functions



Note: This figure shows how different values of the elasticity of capital substitution determine the functional forms of firm's production functions. The range of the elasticity of capital substitution is from zero to infinity.

capital misallocation literature

$$Y_i = A_i \left(\sum_{m=1}^{M} K_{mi} \right)^{\alpha} L_i^{1-\alpha} \equiv A_i K_i^{\alpha} L_i^{1-\alpha}. \tag{2.3}$$

This is a simple two-factor Cobb–Douglas production function, where we define the total capital stock of firm i as $K_i = \sum_{m=1}^{M} K_{mi}$. In the data, K_i is just the total fixed assets, corresponding to the total book value of different types of capital stock, after normalizing all types of capital stock as dollar-valued unit.[9]

Labor and all M types of capital have a fixed aggregate supply such that $\sum_{i=1}^{N} L_i = L$ and $\sum_{i=1}^{N} K_{mi} = K_m, \forall m$. Firms face $M+1$ types of distortions.[10] The first type, as output distortion τ_Y, increases firm's marginal products of all types of capital and labor by the same proportion. The rest of M types of distortions, $\{\tau_{Kmi}\}_{i=1}^{M}$, raises marginal products of type M-th capital relative to labor. The profit maximization problem of the firm is given by:

$$\max_{\{P_i, Y_i, K_{mi}, L_i\}} (1+\tau_{Yi}) P_i Y_i - \sum_{m=1}^{M} (1+\tau_{Kmi}) R_m K_{mi} - W L_i,$$
$$\text{subject to } : Y_i = Y \left(\frac{P}{P_i} \right)^{\sigma}.$$

The details of solving the firm's problem is attached in Appendix A.1.[11] In equilibrium,

[9] Since what we observe from the data is always the nominal value of different types of capital, we can normalize their quantities so that their prices are all equal to one. In the capital misallocation literature, the prices of factors do not impact the measurements, since, intuitively, only the within-sector second moment of factors' efficiency matters for the measurements. We will introduce this unit of choice more formally in the data section.

[10] Because there are $M+1$ factors of production, we can only separately identify $M+1$ distortions. Here, we keep our framework consistent with HK09, modeling all distortions as the distortions relative to labor.

[11] To quickly see how a capital-specific wedge distorts the usage of it, see the firms' first-order conditions with respect to capital K_{mi}:

$$\frac{K_{mi}}{\left(\sum_{n=1}^{M} \alpha_n^{\frac{1}{\gamma}} K_{ni}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}} = \alpha_m \left[\frac{R_m (1+\tau_{Kmi})}{\left(\sum_{n=1}^{M} \alpha_n R_n^{1-\gamma} (1+\tau_{Kni})^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right]^{-\gamma}, \tag{2.4}$$

where the optimal use of type-m capital, K_{mi}, relative to others, is influenced by its cost relative to other capital costs. A larger distortion τ_{Kmi} discourages a firm from using K_{mi}. Additionally, when facing the same τ_{Kmi}, a higher γ value further disincentivizes the use of type- m capital as it allows for easier substitution the marginal revenue products of capital m and labor L_i can be written as:

$$MRPK_{mi} = \frac{1+\tau_{K_{mi}}}{1+\tau_{Y_i}} R_m, \quad MRPL_i = W \frac{1}{1+\tau_{Y_i}} \tag{2.5}$$

and the intuition is straightforward: the marginal revenue product of capital type m is increasing in the type-specific distortion. Firms with a larger distortion, i.e., higher τ_{Kmi}, will find it more expensive to use K_m and will use an insufficient amount in production, yielding a higher revenue return than the market rental rate.

The dispersion of τ_{Kmi} among firms indicates misallocation: there could be an output gain by reallocating capital K_m from firms with lower $MRPK_{mi}$ to those with higher $MRPK_{mi}$.[12] However, it is worth mentioning that with a CES production function in a distorted economy, there does not exist a closed-form sectoral aggregation production function. This prevents us from using the log-normal assumption and the variance of $MPRK_{mi}$ dispersion as sufficient statistics to infer the cost of capital misallocation in an economy.[13]

It is worth mentioning that, in two special cases, our firm's optimization problem coincides with the firm's problem with homogeneous capital as in the HK09 case. The first situation occurs when the elasticity of capital substitution approaches infinity, as mentioned above. In this case, all types of capital become identical for the firm's production; firms will then, in the short run, purchase only the capital type with the lowest price, and in the long run, only one type of capital will be produced. The second situation occurs when the correlation between distortions of different types of capital is independent of the firm, i.e., $\frac{1+\tau_{Kmi}}{1+\tau_{Kni}} = \beta_{mn}$. In this case, each firm will optimally use the same ratio of different types of capital in production, allowing us to rewrite the CES production function as a Cobb-Douglas production function with only one capital investment wedge.

2.2 Efficient Allocation and Two Measures of Misallocation

To understand the aggregate implications of heterogeneous capital distortions, we characterize the efficient counterfactual level of output when all input distortions are absent with the following lemma

Lemma 1 *The allocation that maximizes the output of the economy satisfies:*

with other capital types in production.

[12] One can think of the capital wedges in our model as shadow prices representing the constraints firms encounter in different asset markets. These constraints could take the form of collateral restrictions when renting equipment or buildings and thus limit firms' access to factors of production. The wedges, therefore, capture the economic costs imposed by these constraints and the resulting distortions in resource allocation. In any case, the wedges summarize the inefficiencies arising from distorted factor markets and the resulting deviations from the optimal allocation of resources.

[13] When $\gamma = 1$ as a special case, we can still use the variance of marginal revenue products of different types of capital as sufficient statistics to infer the cost of misallocation. In a special case with Cobb-Douglas function, $MRPK_{mi}$ is the same as average revenue product of capital ($ARPK_{mi}$) which is defined as sales over capital K_{mi}.

$$L_i^e = \frac{A_i^{\sigma-1}}{\sum_{j=1}^{N} A_j^{\sigma-1}} L, \tag{2.6}$$

$$K_{mi}^e = \frac{A_i^{\sigma-1}}{\sum_{j=1}^{N} A_j^{\sigma-1}} K_m, \quad \forall m \tag{2.7}$$

for all firms, given firm-level productivities A_i and the aggregate factor supply L and K_m for all m. Under this efficient allocation, the efficient level of aggregate output Y^e is determined by the following aggregate production function:

$$Y^e = TFP^e \cdot \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1} \cdot \alpha} \cdot L^{1-\alpha}, TFP^e = \left(\sum_{i=1}^{N} A_i^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{2.8}$$

Proof. See Appendix A.3. ■

Lemma 1 can be derived straightforwardly from the condition that $MRPK_{mi}$ and $MRPL_i$ are the identical across different firms.[14] The conditions for efficient allocation reveal a proportional relationship: a firm with higher productivity gets a larger share of labor and each type of capital. Moreover, the economy features a Cobb-Douglas aggregate production function. The aggregate efficient TFP is a CES aggregated firm-level productivity.

With the efficient benchmark established, we can define a measure of distance to the frontier as a gap between actual output and the ideal output described in Lemma 1. This will allow us to characterize the cost of misallocation. Specifically, we will focus on two closely related measures of misallocation that are commonly used in the literature: *allocative efficiency* (e.g. Bils et al. (2021)) and *output loss* (e.g. D. Baqaee and Farhi (2022)).

Definition 1 *Allocative Efficiency (AE) is defined as:*

$$AE = \frac{Y}{Y^e} \tag{2.9}$$

Allocative efficiency captures the proximity of the actual output to the ideal scenario posited in Lemma 1. It is defined as the ratio of the economy's actual output to what might have been achieved in a distortion-free economy. An $AE = 1$ denotes that the economy is operating at its efficient frontier with no misallocation. Conversely, $AE < 1$ highlights the presence of misallocation such that the economy is producing less than its full potential.

Definition 2 *Output Loss ($\mathcal{L}$) is defined as:*

$$\mathcal{L} = \log Y^e - \log Y = -\log(\frac{Y}{Y^e}) = -\log(AE) \tag{2.10}$$

[14] This also implies that $TFPR_i$ is identical across all firms instead of TFP_i. Here, we use the same definition in Hsieh and Klenow (2009) which defines $TFPR_i$ as the revenue productivity, and TFP_i or $TFPQ_i$ as the (quantity) productivity. In this framework $TFPR_i \equiv \frac{\sigma}{\sigma-1} \left[\frac{\left(\sum_{m=1}^{M} \alpha_m MRPK_i^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}{\alpha} \right]^{\alpha} \left(\frac{MRPL_i}{1-\alpha} \right)^{1-\alpha}$.

Output loss measures the actual loss in output due to misallocation using a logarithmic scale to show the percentage difference from the efficient output level. We provide a simple example with a Cobb-Douglas economy as a special case to better understand how input distortions affect allocative efficiency in Appendix A.2.

2.3 Measurement Primitives and Conventions

When applying our measurement framework to datasets in order to quantify misallocation from the data, three measurement primitives are needed: firm-level dataset ($\mathcal{D}$), fixed parameters ($\mathcal{P}$), and a choice of elasticity of capital substitution (γ).[15] The firm-level dataset includes revenue (P_iY_i), wage bills (L_i) and capital stocks of different types ($\{K_{mi}\}_{m=1}^{M}$).[16] The fixed model parameters contains the elasticity of goods substitution (σ), the capital share (α) and the CES share parameters capturing the relative importance of different types of capital in the capital bundle ($\{\alpha_m\}_{m=1}^{M}$).

Bringing data to our framework, we specify the unit for any type m of capital, K_m, as the dollar-valued quantity of that capital. This allows us to follow the common practice in misallocation literature, taking the book value of capital (e.g., PPEGT/PPENT in Compustat) as the capital stock in the firm. Moreover, when breaking down the capital into its types, the stock values are comparable as they are all valued in current-dollar terms. Therefore, K_m can refer to both the quantity of K_m in the model and its market (book) value in dollars in the data.

More specific to the fixed parameters calibration, we assume that researchers calibrate the CES share parameters according to Convention 1, as shown below.

Convention 1 *Under the choice of unit, the CES share parameters are measured as the share of market (book) value:* $\alpha_m = \frac{K_m}{\sum_m K_m}$.

We argue that convention 1 is consistent with the empirical norm of calibration. From a calibration point of view, as suggested by Klump, McAdam, and Willman (2012), the normalization of CES share should be in correspondence to the expenditure share in the reference economy. However, since the expenditure share is almost never observed in the literature using the CES aggregator on different assets, researchers commonly use the share of dollared stock value to calibrate the CES share of an asset, as in Whited and Zhao (2021).

We also think that convention 1 is theoretically consistent. Consider the net rental rate of the capital of type m (given the static nature of our accounting model) , R_m, and the nominal price of the capital of type m, P_m^K. As noted by Jorgenson (1963), Karabarbounis and Neiman (2019), and recently in vom Lehn and Winberry (2022), the net rental rates

[15] The elasticity of capital substitution, γ, is the parameter central to our analysis. In the process of misallocation measurement, researchers might assume different values of γ. We will take the data and fix other model parameters, but provide comparative statics of measured misallocation by varying γ. Thus, we can provide quantitative statements on the size of measured misallocation between a researcher choosing $\gamma = \infty$ another one choosing a finite γ.

[16] For example, Compustat provides data on book values of various capital assets like Machinery and equipment (FATE), Buildings (FATB), and others. By mapping these assets to our theoretical types of capital, we can obtain measures of K_{mi}.

of type m at time t, measured without aggregate taxes or wedges, is given by: $\frac{R_{mt}}{P^K_{mt}} = \left[\left(\frac{P^K_{mt-1}}{P^K_{mt}}\right)(1+r_t) - 1\right]$, where r_t is the aggregate interest rate. Then, in a static model where the steady state behavior of the pricing model is analyzed, we would have that for any two assets m and n: $\frac{R_m}{P^K_m} = \frac{R_n}{P^K_n}$. With our choice of quantity unit, the price unit would be $P^K_m = P^K_n$, which implies that $R_m = R_n$. Therefore, the net sectoral expenditure share of capital in our static model would be:

$$\alpha_m = \frac{R^\gamma_m K_m}{\sum_n R^\gamma_n K_n} = \frac{K_m}{\sum_n K_n}, \tag{2.11}$$

which is consistent with Convention 1.

2.4 Main Theoretical Results: Measured Misallocation with Capital Heterogeneity

Given the same dataset and fixed model parameters, our goal is to demonstrate that within this framework, the larger the elasticity of capital substitution considered by researchers, the smaller the measured cost of capital misallocation. To see this, we first show in the following lemma that assuming different values of γ affects the measurement of firm-level productivity.

Lemma 2 *Conditional on the same data and fixed model parameters, measured firm-level TFP will decrease with the chosen elasticity of capital substitution*

$$\frac{\partial A_i(\gamma)}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left[\kappa \frac{(P_iY_i)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{m=1}^M \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha}}\right] \leq 0. \tag{2.12}$$

Proof. See Appendix A.4. ■

The proof is in the spirit of the results in Klump and de La Grandville (2000). κ is a constant which is irrelevant to firm's productivity.[17] The implication of this lemma is quite straightforward: a larger γ suggests greater flexibility in substituting one type of capital for another. Hence, for a given set of inputs, higher γ will measure a higher level of input bundle $\left(\sum_{m=1}^M \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha}$. Therefore, given the same observed revenue P_iY_i in the data, a lower level of firm-level TFP will be measured for researchers.[18]

[17] The scalar $\kappa = W^{1-\alpha}(PY)^{-\frac{1}{\sigma-1}}/P$, where W, Y and P are wage, aggregate output and price respectively. In the dataset, we do not observe aggregate output and price seperately from the data. However, since both firm-level output and price will not change with different elasticity of capital substitution, so κ can be normalized as one. This just says that if with a good dataset where we can get information of firm-level output and price seperately, it won't affect our results.

[18] Notice that our framework has no restrictions on the capital wedges, $\tau_{K_{mi}}$. The only factor we consider is capital's imperfect substitutability. In fact, when the capital wedges across different types of capital are identical, i.e., there are no cross-type capital distortions, the capital distribution across different types of capital within a firm is the same across different firms. Hence, in this case, our Convention 1 becomes $\alpha_m = \frac{K_m}{\sum_m K_m} = \alpha_{mi} = \frac{K_{mi}}{\sum_m K_{mi}}$. This means that the CES share calibration happens to be firm-specific. In

The insight from Lemma 2 will be the heart of our arguments for the main results. When a firm's technology inherently permits minimal substitution (i.e., small γ), but a researcher assumes the capital types to be highly substitutable, the resulting measurements can overstate the firm's productivity, leading to an inaccurately smaller production possibility frontier. This overestimation of productivity will affect the estimation of the counterfactual first-best output and ultimately alter the results of capital misallocation, as demonstrated in the following proposition.

Proposition 1 *Conditional on the same data and fixed model parameters that satisfy Convention 1, we always have:*

$$\gamma < \gamma' \Rightarrow AE(\gamma) < AE(\gamma'). \tag{2.13}$$

Proof. See Appendix A.5. ■

When comparing allocative efficiency between two measurements with different γ, we are essentially comparing two different first-best outputs, as the observed output is based on the data. Recall from Lemma 1 that the efficient aggregate output is composed of aggregate productivity and a capital bundle. When Convention 1 holds, the CES capital bundle collapses into the sum of all different types of capital, which is independent of γ. Hence, γ only affects the efficient output through the productivity component.[19]

The insight from Proposition 1 is that when researchers think firms can more easily replace one type of capital with another, they perceive firms as less productive (from Lemma 1). Therefore, when input sets are fixed, a larger γ results in a smaller production possibility frontier, reducing the efficient level of output and increasing measured allocative efficiency.

When further disaggregating different types of capital into more detailed categories, we observe that finer disaggregation of capital inputs also leads to lower measured efficiency, as shown below

Corollary 1 *For any datasets* $\mathcal{D} = (R_i, L_i, \{K_{mi}\}_{m=1}^{M})_{i=1}^{N}$ *and* $\mathcal{D}' = (R_i, L_i, \{K_{m'i}\}_{m'=1}^{M'})_{i=1}^{N}$ *where* $M' > M$*, and model parameters* $\mathcal{P} = (\sigma, \alpha, \{\alpha_i\}_{i=1}^{M})$ *and* $\mathcal{P}' = (\sigma, \alpha, \{\alpha_{i'}\}_{i'=1}^{M'})$ *that satisfy Convention 1:*

$$AE(\gamma, \mathcal{D}', \mathcal{P}') \leq AE(\gamma, \mathcal{D}, \mathcal{P}).$$

Proof. See Appendix A.6. ■

In other words, further disaggregation of capital into finer types will measure more misallocation, due to the fact that disaggregation reveals more about the previously unaccounted imperfect substitutability of capital, which amplifies the cost of underlying distortions. This proposition suggests that the more detailed capital category researchers observe from datasets, the larger cost of capital misallocation they will measure using our framework.

this case, $\frac{\partial A_i(\gamma)}{\partial \gamma} = 0$.

[19] However, in the empirical measurement we will show that violation of Convention 1 will not qualitatively change the results.

Our results still hold when we observe actual capital expenditure in the dataset. To see this, let $R_m K_m$ for $m = 1, 2, \ldots, M$ represent the expenditure on type-m capital. The CES share is then calibrated as $\alpha'_m = \frac{R_m K_m}{\sum_m R_m K_m}, \forall m$, which is equivalent to normalizing type-m capital K_m by R_m. Consequently, when inferring the firm's productivity, we use $\frac{K_{mi}}{R_m}$, the normalized unit of capital. With this adjustment, our proof remains valid.

2.5 Measurements in the Multi-sector Economy

In empirical measurements of capital misallocation, it is important for researchers to consider sectoral heterogeneity. The previous framework was built up on a simple economy, figuring one sector with multiple firms, so we extend it into a multi-sector economy model in this section.

We assume a representative firm producing the final good Y in a competitive market, by combining sectoral output Y_s of S different industries using a Cobb-Douglas production function

$$Y = \Pi_{s=1}^{S} Y_s^{\theta_s}, \text{ and } \sum_{s=1}^{S} \theta_s = 1, \tag{2.14}$$

where θ_s is the share of sectoral expenditure in GDP. Within each sector, the setting is the same as our one-sector framework, but different sectors are different in their capital (for any types) and labor intensities and endowments.

Although it does not affect our quantitative measurement, our theoretical Proposition 1 and Corollary 1 remain valid in a multi-sector economy under the assumption of no cross-sectoral distortions, i.e., we do not allow reallocation across sectors. The market clearing conditions are

$$\sum_{i=1}^{N_m} K_{mi} = K_m, \sum_{i=1}^{N_m} L_i = L_m, \forall m \in \{1, 2, ..., M\}, \tag{2.15}$$

where N_m is the number of firms in different sector. Formally,

Proposition 2 *In a multi-sector economy, for each sector S and correspondent datasets $\mathcal{D}_s$ and model parameters $\mathcal{P}_s$ satisfying Convention 1, Proposition 1 and Corollary 1 hold true when the economy has no sectoral distortions.*

Proof. See Appendix A.7. ■

This focus on within-industry misallocation aligns with the work of Hsieh and Klenow (2009) and Bils et al. (2021),[20] among others, but differs from the approaches taken by Jones (2011), Hang, Krishna, and Tang (2020),[21] and other papers considering production

[20] In a robustness check of Hsieh and Klenow (2009), they show that: *"Cobb-Douglas aggregation across sectors means that TFPR equalization does not affect the allocation of inputs across sectors; the rise in a sector's productivity is exactly offset by the fall in its price index"*. Bils et al. (2021) models the cross secteral distortions but only focus on within sector misallocation.

[21] In the equation of sectoral capital and labor share relative to the total endowment in Hang et al. (2020), if we assume $\mathcal{T}_{Ks}$ and $\mathcal{T}_{Ls}$ are both zero, our Proposition 1 and Corollary 1 can still be applied to their framework.

networks. We stick to only study the within-sector misallocation, considering the fact that reallocation across different sectors on each specific capital might lead to some counter-intuitive situations. For example, the machines in a coffee shop can be reallocated to a body shop. [22] The Cobb-Douglas aggregation across capital bundle and labor in firm's production will not affect our proof (Mallick and Maqsood (2023)).

The reason why cross-sectoral factor reallocation will hurt our theoretical results is that the calibration of CES shares in Convention 1 does not allow the elasticity to be canceled in the aggregate capital bundle. Hence, when comparing two efficient outputs with different elasticities, researchers need to consider whether the effect on the measured aggregate TFP or capital bundle is larger. As a final remark, our framework can still be applied to measure when there are cross sectoral distortions, but whether larger elasticity of capital substitution increases the measured cost of misallocation can only be found quantitatively.

3 Firm-level Data and Inferring Allocative Efficiency

To infer capital misallocation, our framework requires data on firm-level output, labor expenditure, and the book values of different types of capital. We primarily use data from Compustat North America, the Indian Annual Survey of Industries (ASI), and Orbis Global Financials. Table 1 summarizes the geographical coverage, sample period, and types of capital covered in each dataset.

3.1 Three Main Datasets

The Compustat North America database, managed by S&P Global, serves as our primary data source, covering over 10,000 U.S. public firms from 1985 to 2019 and providing detailed financial information on these companies.[23] We use Compustat's classification of fixed assets into two types: structures and equipment. The 'structures' category includes physical constructions such as buildings, facilities, and land improvements, while the 'equipment' category encompasses tangible movable assets like machinery, hardware, vehicles, and tools.[24] After removing outliers and observations with missing data, we are left with a total of 116,143 firm-year observations across 96 3-digit NAICS sectors from 1985 to 2019.

[22] Oberfield (2013) shows that during Chilean crisis of 1982, between industry allocational efficiency accounts for roughly one-third of TFP loss. Syverson (2011) summarizes that even within 4-digit SIC industries in the US, the average difference in logged total factor productivity is still quite large.

[23] In comparison to the Longitudinal Research Database (LRD) used in Bils et al. (2021) or the Census of Manufacturing used in HK09, Compustat has relatively narrower coverage, focusing on publicly traded firms. Nevertheless, it offers detailed yearly book value data on five fixed asset categories for most firms, which meets our requirements.

[24] We have chosen to adopt a more aggregated approach to equipment and structure delineation, as opposed to utilizing Compustat's detailed classification into four or five granular capital categories. This decision stems from the dataset's coverage of approximately 5,000 firms initially, with many of them not reporting expenditures across various fine-grained asset classes. While limited in granularity, this extensive dataset establishes a useful comparative benchmark of capital allocation patterns.

Table 1: Data Sources with Firm-level Capital Allocations

Data Source	Coverage	Years	Types of Capital
Compustat	United States	1985-2019	1. Structure 2. Equipment
Indian ASI	India	2000-2019	Structure: 1.1 Land 1.2 Building Equipment: 2.1 Plant+Machinery 2.2 Transportation 2.3 Computer 2.4 Others
Orbis (Detailed Format)	China, Japan, UK, Canada, Australia	2010-2019	1. Equipment 2. Structure 3. Others

Note: This table shows the datasets used in our empirical and quantitative exercise. The Compustat North American is from WRDS database. Indian ASI is from Indian Ministry of Statistics and Programme Implementations. The Orbis Detailed Format is from Oribs Global.

Our data for India comes from the Annual Survey of Industries (ASI).[25] ASI conducts a survey that ensures national representativeness of formal manufacturing plants in India, with coverage extending to plants employing a minimum of 10 workers with power usage and a minimum of 20 workers without power usage.[26] It covers the universe of large plants and a random but representative sample of small plants in India's manufacturing sector. ASI consistently covers six types of capital inputs, which can be broadly categorized into equipment and structure. Structures encompasses land and buildings, while equipment includes plant and machinery, transportation equipment, computers, and other miscellaneous equipment. We use the official panel identifiers to link the longitudinal surveys into a panel database, and we harmonize all sector classifications into 3-digit NIC-98 classification. In the full sample, we have a total of 701,543 firm-year observations across 95 sectors. We also use a restricted sample, in which firms use all six types of capital inputs, covers 316,863 firm-year observations spanning 89 sectors.

The third dataset we use is the Orbis Global Financials dataset (Detailed Format), which provides firm-level financial information across multiple countries from 2010–2019.[27] Specifically, Orbis covers companies in China, Japan, the UK, Canada, Australia, and several other European nations. This dataset categorizes capital expenditures into three broad categories: equipment, structures, and other. The equipment category includes machinery, vehicles, hardware, and other tangible operational assets. Structures encompasses buildings, facilities, and land. The "other" category captures miscellaneous fixed assets not classified as either equipment or structures. The "Detailed Format" Orbis data covers samples of firms in several industrialized economies, and is used as a robustness check.[28]

[25] ASI has been used by many researchers to studied capital misallocation and growth (see, for example, Hsieh and Klenow (2009), Bils et al. (2021), and Boehm and Oberfield (2020)).

[26] Another advantage of ASI is that the information is at plant-level instead of firm-level. Kehrig and Vincent (2021) argued that, for multi-plant firms, firm-level marginal products are not a good proxy when measuring the loss of misallocation

[27] Others who studied capital misallocation and used Orbis as the main data source include Gopinath et al. (2017), Jurzyk and Ruane (2021) and so on.

[28] A key benefit of Orbis is providing an international perspective on asset-specific capital misallocation, com-

The main variables we use include revenue, labor and different types of capital. In Compustat, we measure a firm's output using sales (SALE)[29], labor as number of employees (EMP), equipment (FATE) and structures as total fixed assets (PPEGT) minus equipment (FATE). In ASI, we construct the revenue as total sales, labor as total wage bill of all workers, material cost as total input costs and six types of capital as the net value of lands, buildings, plants and machinery, transportation, computers and others (fixed assets minus the first five types), and labor as wage bills. The details of variable construction and data cleaning are attached in Appendix B.

3.2 Inferring Allocative Efficiency

We use $\wedge$ to denote measued values from datasets. We follow the literature to measure TFPR and TFPQ as

$$TFPR_{si} \equiv \frac{\widehat{P_{si}Y_{si}}}{\left(\sum_{m=1}^{M} \alpha_{ms}^{\frac{1}{\gamma}} \widehat{K}_{msi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} \widehat{L}_{si}^{1-\alpha}}, \tag{3.1}$$

$$TFPQ_{si} \equiv \kappa_s \frac{\left(\widehat{P_{si}Y_{si}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{m=1}^{M} \alpha_{ms}^{\frac{1}{\gamma}} \widehat{K}_{msi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} \widehat{L}_{si}^{1-\alpha}}, \tag{3.2}$$

and we normalize κ_s as 1 when we only consider within-sector factors reallocation. We use firm revenue to infer the realized real sectoral aggregate output

$$\widehat{Y}_s = \left(\sum_{i=1}^{N} \widehat{Y}_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\sum_{i=1}^{N} \left(\left(\widehat{P_{si}Y_{si}}\right)^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\sum_{i=1}^{N} \widehat{P_{si}Y_{si}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{3.3}$$

then we use measured $TFPQ_{si}$ to determine what are the optimal distribution of factors across firms in sector s, following Lemma 1

$$\widehat{L^e}_{si} = \frac{TFPQ_{si}^{\sigma-1}}{\sum_{j=1}^{N} TFPQ_{sj}^{\sigma-1}} \widehat{L}_s, \tag{3.4}$$

$$\widehat{K^e}_{msi} = \frac{TFPQ_{si}^{\sigma-1}}{\sum_{j=1}^{N} TFPQ_{sj}^{\sigma-1}} \widehat{K}_{ms}. \quad \forall m \tag{3.5}$$

Using the inferred counter factual capital and labor distribution, we measure the counter

plementing our focus on US and India. However, inconsistencies in reporting standards between countries requires caution in making direct cross-country comparisons.

[29] Value added measures are not available in Compustat

factual firm output and sectoral output

$$\widehat{Y^e}_{si} = TFPQ_{si} \left(\sum_{m=1}^{M} \alpha_{ms}^{\frac{1}{\gamma}} \widehat{K^e}_{msi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} \widehat{L^e}_{si}^{1-\alpha}, \tag{3.6}$$

$$\widehat{Y^e}_s = \left(\sum_{i=1}^{N} \widehat{Y^e}_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{3.7}$$

Finally, we can measure the allocative efficiency by comparing the ratio of two aggregate outputs

$$\widehat{AE} = \frac{\widehat{Y}}{\widehat{Y^e}} = \frac{\prod_{s=1}^{S} \widehat{Y}_s^{\theta_s}}{\prod_{s=1}^{S} \widehat{Y^e}_s^{\theta_s}} = \prod_{s=1}^{S} \widehat{AE}_s^{\theta_s}. \tag{3.8}$$

To calibrate the measurement framework, we set the country-sector capital shares (α_s) by the median value of cost shares, following Asker et al. (2014).[30] We also set country-sector output shares (θ_s) by the ratio of sectoral and economy-wise value-added. For the CES shares of heterogeneous types of capital, we set α_m corresponding to the share of each type of capital following Convention 1. We set the elasticity of substitution between goods $\sigma = 6$. [31]

4 Estimation of the Elasticity of Capital Substitution

4.1 Estimation Approach

We follow Caunedo, Jaume, and Keller (2023b) to estimate the elasticity of capital substitution γ using the first order conditions from the firm's maximization problem. The CES structure yields the relative demand curve between the quantity ratio and the price ratio of capital m and m',

$$\ln \left(\frac{K_{sit}^{m}}{K_{sit}^{m'}} \right) = \gamma \ln \left(\frac{R_{st}^{m'}}{R_{st}^{m}} \right) + \gamma \ln \left(\frac{(1+\tau_{sit}^{m'})}{(1+\tau_{sit}^{m})} \right) + \gamma \ln \left(\frac{\alpha_{ms}}{\alpha_{m's}} \right). \tag{4.1}$$

Estimating with this FOC, one needs to detailed data for both capital stock at the firm-level and rental rate at the sector level. The only data source that could provide us with such variations is the US Compustat data, accompanied with the detailed fixed asset prices from BEA Fixed Asset Table (FAT).[32] We thus benchmark reduced-form exercises to have only two types of capital: equipment and structure from Compustat. We work with the

[30] David and Venkateswaran (2019) and Gopinath et al. (2017) set a constant factors shares across all sectors, but their focus is on the structure part.

[31] The macroeconomics literature estimates this number as from 3 to 10. Hsieh and Klenow (2009) uses 3, Bils et al. (2021) uses 4 and David and Venkateswaran (2019) uses 6.

[32] The website of BEA Fixed Asset Table (FAT) is: https://www.bea.gov/itable/fixed-assets.

following empirically specification of (4.1),

$$\ln\left(\frac{S_{sit}}{E_{sit}}\right) = \text{Conts} + \gamma \ln\left(\frac{R^E_{st}}{R^S_{st}}\right) + \text{FEs} + \varepsilon_{it}, \tag{4.2}$$

where S_{sit} and E_{sit} are the firm-level capital stock constructed from the perpetual inventory method; R^E_{st} and R^S_{st} are the sector-specific equipment and structure user costs. FEs include firm fixed-effect and year fixed-effect, and Conts refers to constants, such as factor shares ratio. Our goal is to estimate an unbiased γ which to be used in measuring capital misallocation.

4.2 Data and Construction of User Costs and Capital Quantities

Data Every year, the Bureau of Economic Analysis (BEA) publishes detailed data on the nominal value of capital stock for over 50 categories of detailed assets owned by each 3-digit industry in the Fixed Asset Table (FAT). Furthermore, all tangible assets are classified into equipment or structures. Price indexes for private fixed investment of these assets are provided in the NIPA Table of BEA. For assets in structures, we use the BEA prices; however, for equipment assets, we instead use the Cummins and Violante (2002) quality-adjusted prices as in Caunedo et al. (2023b).

Sector-level User Cost of Equipment and Structures. The R^E_{st} and R^S_{st} are the sector-year specific user costs of equipment and structures. To measure these, we first construct an asset-specific user cost for all the subcategories (30 types in total) of equipment ($\{E\}$) and structures ($\{S\}$) in the BEA fixed asset table using the Jorgenson (1963)'s user cost formula,[33]

$$R_{at} = \frac{P_{at-1}}{\lambda^c_{t-1}}\left[R - \left(1 - \bar{\delta}_{at}\right)\frac{\frac{P_{at}}{\lambda^c_t}}{\frac{P_{at-1}}{\lambda^c_{t-1}}}\right],$$

where a is a detailed asset type, $\bar{\delta}_{at}$ is the average depreciation rate and P_{at-1} is its price index. R is the risk-free rate and λ^c_t is the consumption price index.[34] The BEA provides nominal capital stock for detailed assets at the 3-digit NAICS level, which allows us to compute an aggregate sector-year level rental rate for equipment (structures) capital using a Törnqvist index over R_{at} using a's expenditure share in the total equipment (structures) user costs as aggregation weights,

$$\ln(R^X_{st}) = \sum_{a\in\{X\}} \frac{\hat{R}_{at}P_{sat}K_{sat}}{\sum_{a\in\{X\}}\hat{R}_{at}P_{sat}K_{sat}} \times \ln(R_{at}), \quad \forall X = E, S, \tag{4.3}$$

[33] This is defined as the real user cost per unit of real asset stock.

[34] The rationale of the formula is the following: form an investor's perspective, the post-depreciation capital gain of any asset, when evaluated in the unit of consumption goods, must satisfy the no-arbitrage condition and equal to the risk-free rate. Alternative measures of R include the annual real interests rate or the AAA bond yield (to account for the risk-premium). We have verified that our estimates for γ is not sensitive to the choice of R.

where $\hat{R}_{at} = \frac{R_{at}\lambda^c_{t-1}}{P_{at-1}}$ is the user cost per dollar of asset a and $P_{sat}K_{sat}$ is the measured nominal capital stock of asset a in sector s. Therefore, $\frac{\hat{R}_{at}P_{sat}K_{sat}}{\sum_{a\in\mathcal{E}}\hat{R}_{at}P_{sat}K_{sat}}$ is the cost share of a out of all assets in equipment.

Capital Quantities of Equipment and Structures We apply the standard perpetual inventory method to construct the quantity of equipment and structure at the firm level in Compustat. We observe the book values of equipment and structures at the firm-year level and treat the depreciation-adjusted differences of book values between years as the nominal value investment.

The main challenge lies in the absence of sector-specific investment goods price for equipment and structures at the sector level. To address this, we use the detailed asset prices P_{at} and their nominal investment expenditure $P_{sat}I_{sat}$ at the sector-year level from FAT to construct a Tornqvist Index of equipment and structure investment goods prices

$$\ln(P^{I_E}_{st}) = \sum_{a\in\{X\}} \frac{P_{sat}I_{sat}}{\sum_{a\in\{X\}} P_{sat}I_{sat}} \ln(P_{at}), \quad \forall X = E, S. \tag{4.4}$$

Using the constructed investment price series, we first initialize the equipment stock of each firm as $E_{i0} = \frac{\text{Book Value}^E_{i0}}{P^{I_E}_{i0}}$ using the firm's first observation in the panel and then compute the investment quantity in year t as $I^E_{it+1} = \frac{\text{Book Value}^E_{it+1} - (1-\delta_{st})\text{Book Value}^E_{it}}{P^{I_E}_{it}}$. Finally, we can iteratively compute firm i's equipment stock as $E_{it+1} = (1-\delta_{st})E_{it} + I^E_{it+1}$. The construction for structures stock follows the same logic.

4.3 Estimation Results

Applying Equation 4.1 to Compustat, we exploit the relative rental rate differences across 3-digit NAICS sectors in the US in order to identify the firm-level elasticity of substitution between equipment and structure. The inclusion of two-way fixed-effects in our specification ensures that we are only capturing the within-firm and within-year response of input adjustments in reaction to price change. Therefore, the specification in Equation 4.1 identifies the short-run response to a transitory relative price shock. We will also examine the long-run elasticity of capital substitution following the approach in Boehm and Oberfield (2020), detailed in Appendix C.

To account for the potential endogeneity of the relative price of capital to the users' technological differences and distortions, we follow similar strategies as Hubmer (2023) and Castro-Vincenzi and Kleinman (2022) and employ a set of "shift-share" instruments, $\ln(R^{E,IV}_{it}) - \ln(R^{S,IV}_{it})$, for the relative price of capital, where the variables $\ln(R^{E,IV}_{it})$ and $\ln(R^{S,IV}_{it})$ are defined as

$$\ln(R^{X,IV}_{st}) = \sum_{a\in\{X\}} \frac{\hat{R}_{a,1980}P_{sa,1980}K_{sa,1980}}{\sum_{a\in\{X\}}\hat{R}_{a,1980}P_{sa,1980}K_{sa,1980}} \times \ln(P_{at}), \quad \forall X = E, S. \tag{4.5}$$

Each of them is a sector-specific weighted average of the log price of the detailed equipment/structures assets, using the assets' cost shares in industry s in 1980 as weights. In contrast to the sector-level rental rate in Equation 4.3, this shift-share measure leverages both the heterogeneous changes in the price of asset during the 1984-2016 period and the differential exposure at baseline of each industry to these detailed assets.

Table 2: Main Estimation Results of Equipment-Structures Elasticity

	OLS	IV	IV	IV	IV
$\ln\left(\frac{R^E_{st}}{R^S_{st}}\right)$	0.176***	0.313***			0.156***
	(0.024)	(0.074)			(0.030)
$\ln\left(\frac{P^{K_E}_{st-1}}{P^{K_S}_{st-1}}\right)$			0.278***		
			(0.072)		
$\ln\left(\frac{P^{I_E}_{st-1}}{P^{I_S}_{st-1}}\right)$				0.343***	
				(0.075)	
$\ln\left(\frac{S_{sit-1}}{E_{sit-1}}\right)$					0.621***
					(0.011)
$\ln\left(\frac{S_{sit-2}}{E_{sit-2}}\right)$					0.038***
					(0.008)
Firm FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Observations	81,884	81,287	80,853	81,447	60,090
R^2	0.826	0.001	0.007	0.008	0.455
K-P F-stats		527.070	2707.560	1671.604	329.132

The validity of our instrument is rooted in the long-standing literature to explain the three-fold decline of equipment prices since 1960 (Jones (2016)), which has been overwhelmingly regarded as a consequence of of the technological progress of the equipment producers, which is exogenous to the shifting demand from firms. Similarly, there has been a two-fold rise of structure prices and residential structure prices since 1960s. Although less studied, this aggregate trend has been attributed to the supply constraints Rognlie (2016), rather than demand-side factors. Reassuringly, we show in Appendix C that our results are unchanged by only using the SSIV for equipment or for structures.

The estimated results are shown in Table 2. We use three different specifications by choosing: user costs ratio, capital bundle price ratio and investment price ratio as different empirical proxies for the relative price of equipment and structure. Through all specifications, we find that all IV estimates favor an elasticity of capital substitution of 0.3, with our preferred estimates of 0.31 in Column (2). We find that in our sample of Compustat firms, equipment and structure are gross complements in production, which is far from the $\gamma \to \infty$ assumption in the misallocation literature. Interestingly, we have also estimated the long-run elasticity of capital substitution in Appendix C and found the long-run elasticity to be around 0.3 as well. Therefore, we set $\hat{\gamma} = 0.3$ in our preceding analysis.

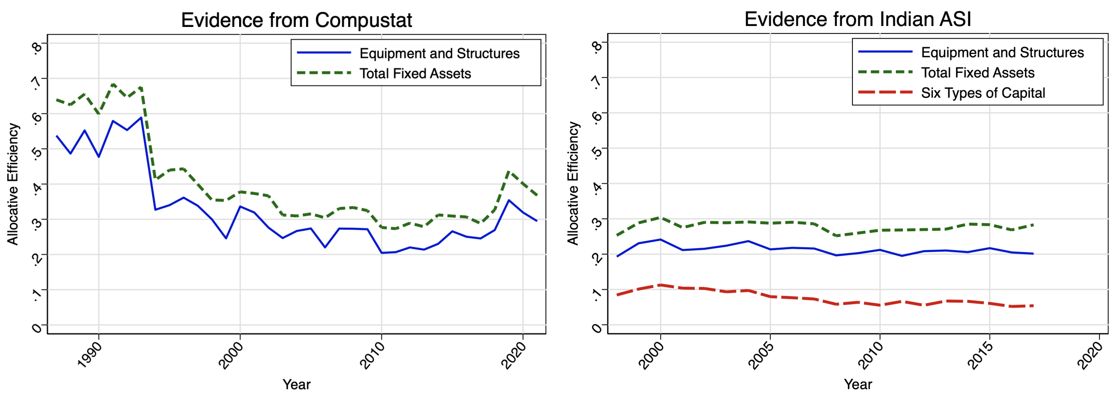
5 Measuring Misallocation under Capital Heterogeneity in Data

Applying datasets to our measurement framework, our objectives are manifold. First, we empirically quantify the magnitude of difference between measuring capital misallocation using total fixed assets and multiple different types of capital. Second, we explore how sensitive this difference is to different values of the elasticity of capital substitution and levels of capital disaggregation.

5.1 Homogeneous Capital vs. Heterogeneous Capital

Using data from Compustat and the Indian ASI, we compute allocative efficiency for each economy every year under two different parameterizations: (1) using equipment and structures with a elasticity $\gamma = 0.3$, our benchmark; and (2) total fixed assets with infinite elasticity ($\gamma \rightarrow \infty$), as HK09 does. Figure 2 shows that the measured allocative efficiency is consistently lower for models with heterogeneous capital in the US Compustat. Not accounting for capital heterogeneity leads to an underestimation of the cost of capital misallocation with scales range from 3.19 to 13.97 percentage points. The average of this difference is 7.1 percentage points, which is almost 25% equivalently.

Figure 2: Time Series AE in the US and India



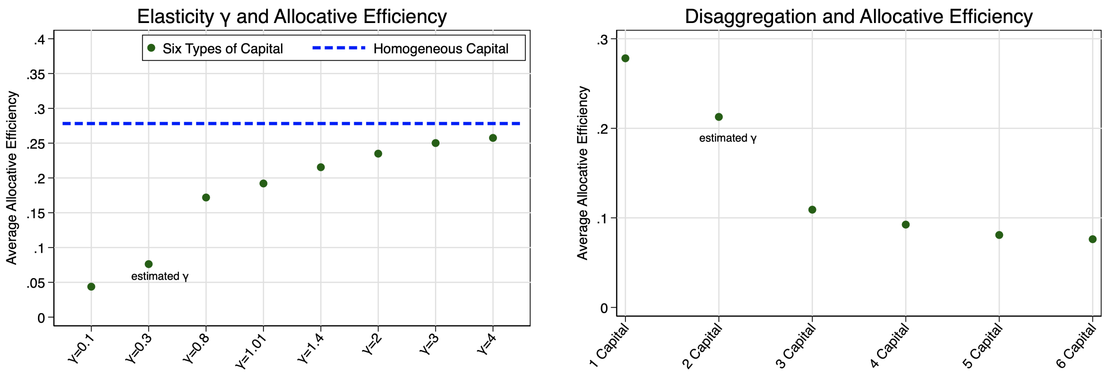
Note: In the figure on the left hand side, we use data from the ASI to measure two allocative efficiency. The allocative efficiency is defined by real output over optimal output. The green line denotes using homogeneous capital as HK–09, the blue dashed line with red dots uses six different types of assets, plant and machinery, transportation, computer, land, building and others, with varying value of γ.

Figure 2 displays similar measurement results for India. We assume that elasticity of substitution between equipment and structures is also 0.3 in India. The difference in allocative efficiency between these using total fixed assets and both equipment and structures to measure ranges also from around 6 to 14 percentage points, which are 14% to 32% of the total welfare loss equivalently. Moreover, we also plot the allocative efficiency measured using six different types of capital (land, building, equipment and machinery, transportation, computer and others). When taking the same number of γ, we measure a very low allocative efficiency, ranging from 2% to 5%. This will indicate that the difference between one and six types of capital can be up to 45% percentage points. These results show that accounting for a $\gamma = 0.3$ elasticity of capital substitution can measures a significantly larger capital misallocation compared to the homogeneous capital case.

5.2 Measured Misallocation and the Elasticity of Capital Substitution

For testing the sensitivity of the measured allocative efficiency to the value of γ, we adopt γ with multiple different numbers from 0.1 to 4 and compute the yearly average allocative efficiency in the left panel of the Figure 3.

Figure 3: Empirical Results of 6 capital from India ASI



Note: In the figure on the left hand side, we use data from the ASI to measure two allocative efficiency. The allocative efficiency is defined by real output over optimal output. The green line denotes using homogeneous capital as HK–09, the blue dashed line with red dots uses six different types of assets, plant and machinery, transportation, computer, land, building and others, with varying value of γ.

Consistent with our theoretical results, the greater the value of γ, the larger the allocative efficiency researchers will measure. When $\gamma = 0.1$, different types of capitals are almost Leontief in production with an extreme amount of complementarity. The measured allocative efficiency is only around 5%, indicating a large loss of misallocation around 95% of the efficient output. In this case, ignoring capital heterogeneity overstates allocative efficiency by roughly 40%. When we set such that different types of capitals are combined using a Cobb-Douglas function in production ($\gamma = 1.01$ as a proxy), the measured AE is 37.35%, and ignoring capital heterogeneity would overstate allocative efficiency by 8.70%. However, as γ becomes very large and different types of capitals are highly substitutable in production, the measured allocative efficiency will converge to the model in which we assume homogeneous capital. Overall, we find that the model with homogenous capital reports only a lower bound on the measured cost of misallocation. Such underestimation will be larger for economies or sectors with strong complementarities between various capital types.

In the right panel of Figure 3, we leverage the detailed disaggregation of capital to measure to what extend the finer disaggregation of capital would measure less allocative efficiency. We measure the disaggregation of total fixed assets into 2 types of capital (equipment and broadly defined structure); 3 types of capital (equipment and structure and others); 4 types of capital (equipment, land, buildings, and others); 5 types of capital (transportation equipment, PP&E, land, buildings and others); and 6 types of capital in our benchmark case.

The results show that measured allocative efficiency with the level of disaggregation. By assuming $\gamma = 0.3$ throughout all cases, we find that, on average, the further disaggregation of one additional capital type led to a decrease of 1.47% measured allocative efficiency. Also, we consistently find that finer disaggregation always leads to more measured misallocation.

5.3 Underestimation of Misallocation in the Global Sample

As a robustness check, we measure the allocative efficiency for other countries using the Orbis data, assuming $\gamma = 1$ (since our Orbis data does not allow us to estimate γ for each countries). Table 3 summarizes the cost of capital misallocation across several countries in percentage terms of potential output. The countries examined include the United States, India, Australia, China, Canada, France, and Japan. For each country, we provide results for different scenarios: one assuming homogeneous capital and the others accounting for heterogeneous capital with different numbers of capital types. Consistent across all countries, the findings reaffirm our theory of measurement. There are substantial disparities between the two approaches in estimated TFP loss, ranging from 2% to 26%.

Table 3: The Undemeasured Costs of Misallocation

	US	**India**	**Australia**	**China**	**Canada**	**France**	**Japan**
Homogenous Capital	38%	52%	36%	37%	19%	13%	13%
Hetero. Capital (2 Types)	43%	54%					
Hetero. Capital (3 Types)			62%	43%	30%	18%	16%
Hetero. Capital (6 Types)		64%					
Undermeasured TFP Loss	5%	2%–12%	26%	6%	11%	5%	3%

Note: This table presents measuring the allocative efficiency across different countries with a Cobb-Douglas production function (i.e. assuming the elasticity of capital substitution is one). Homogeneous capital refers to using the total fixed asset to measure capital misallocation. The first row shows the allocative efficiency using total fixed assets in measurements, and the subsequent three rows are refereed to using different sub-types capital to measure. The last row represents the difference of AE between heterogeneous and homogeneous capital.

5.4 The Decomposition of Misallocation by Capital Types

We now decompose the distortions associated with which types of capital contribute to most of the misallocation costs in the aggregate economy. We characterize the costs of misallocation by applying the D. R. Baqaee and Farhi (2020) results to our measurement framework in the following Lemma.

Lemma 3 *Under Convention 1, in an economy with only capital-related distortions (τ_{Ei} and τ_{Si}), the misallocation costs in the economy, to the second-order, can be expressed as:*

$$\Delta \log TFP = \underbrace{\sum_{s=1}^{S} \theta_s \frac{(\sigma-1)\alpha_s^2\alpha_{Es}^2 + \alpha_s(\alpha_{Es}^2 + \gamma\alpha_{Es}\alpha_{Ss})}{2} Var_\lambda \left[\log(1+\tau_{Esi})\right]}_{\text{Equipment Distortions}}$$

$$+ \underbrace{\sum_{s=1}^{S} \theta_s \frac{(\sigma-1)\alpha_s^2\alpha_{Ss}^2 + \alpha_s(\alpha_{Ss}^2 + \gamma\alpha_{Es}\alpha_{Ss})}{2} Var_\lambda \left[\log(1+\tau_{Ssi})\right]}_{\text{Structures Distortions}}$$

$$+ \underbrace{\sum_{s=1}^{S} \theta_s \alpha_s \alpha_{Es}\alpha_{Ss} \left(1-\gamma+\sigma(\alpha_s-1)\right) Cov_\lambda \left[\log(1+\tau_{Esi}), \log(1+\tau_{Ssi})\right]}_{\text{Mixed Distortions}}.$$

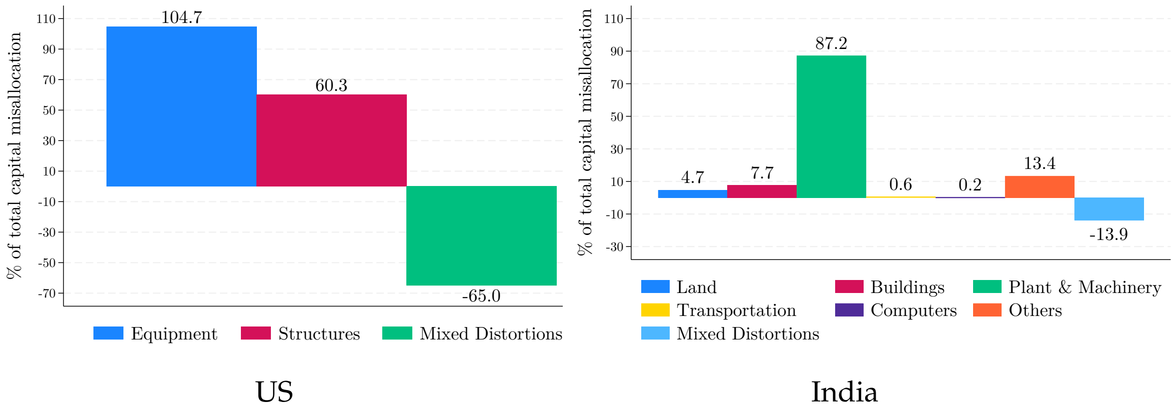
where $Var_\lambda[1+\tau_{Xsi}] = \mathbb{E}_\lambda[(1+\tau_{Xsi})^2] - \mathbb{E}_\lambda^2[1+\tau_{Xsi}]$ is a sales-weighted variance of input distortions and $\lambda_{si} = \frac{P_{si}^e Y_{si}^e}{P_s^e Y_s^e}$ is the sales share of firm i in sector s in the efficient economy.

Proof. See Appendix A.8. ■

Lemma 3 is a generalization of Hsieh and Klenow (2009) with heterogeneous capital and non-unitary elasticity of substitution and without the assumption of log-normality of wedges in the cross-section of firms. From Lemma 3, we see that in the expression of measured TFP loss with structures and equipment capital, $\Delta \log TFP_s$, the first two terms account for the variance in capital distortions associated with equipment and structures, respectively. Moreover, the sales weights take into account the fact that the same distortion is more costly if it is associated with a more productive firm.

A high variance implies a high level of misallocation because firms are not optimally utilizing these types of capital. The last term captures the covariance between the distortions in equipment and structures capital. When $\gamma < 1+\alpha_s(\sigma-1)$, such that different types of capitals are weak substitutes or complements, if these distortions are correlated, say due to a policy that simultaneously taxes factory building and equipment investment for the same firm, then the misallocation (and therefore TFP loss) could be even more severe as it makes the size of productive firms shrink even more. Since the traditional measure using aggregate capital doesn't differentiate between types, it cannot capture the unique misallocation patterns associated with equipment or structure capital.

Figure 4: Decomposition of Aggregate Misallocation on Different Types of Capital



Applying this decomposition formula to the US (2 types of capital) and India (6 types of capital) datasets, Figure 4 show the decomposition of total capital misallocation decomposition for an average year in the sample. In both the United States and India, equipment distortion emerges as the primary source, accounting for 104% and 88% of misallocation costs, respectively. Meanwhile, structure distortion contributes significantly to the misallocation costs in the United States (60.3%), but its impact is comparatively lower in India (12.4%).

Lastly, notice that the "mixed" distortions contribute negatively to the misallocation costs for both the US and India, indicating that different types of capital distortions are, at least on average, negatively correlated across firms. This is a novel finding. Since a firm facing higher-than-market costs for equipment is more likely to face lower-than-market costs for structures, our measurement suggests that considering only those distortions that constrain the use of all types of capital simultaneously is unlikely to provide a complete understanding of capital misallocation.

Notice that not only does capital-specific marginal product dispersion contribute to aggregate capital misallocation, but so does its share in firms' production. In fact, in Compustat, the measured MRPS dispersion is almost three times larger than MRPE dispersion. However, since equipment is more commonly used in production, equipment distortions have a greater impact on the total aggregate cost of capital misallocation.

6 A Firm Dynamics Model with Two Types of Capital

Extending the static measurement framework by explicitly modeling the sources of structures and equipment misallocation in a dynamic context, our goals are twofold: (i) to study which sources contribute to additional misallocation, and (ii) to identify which sources of misallocation explain the efficiency differences between equipment and structures. To estimate our model, we follow and extend the methodology in David and Venkateswaran (2019), which disentangles misallocation into adjustment costs, informational frictions and other firm-specific wedges.

6.1 Model Set-up: Extending the Static Framework

Environment Time is discrete and infinite. A representative household living in the economy consumes the final good, and provides labor of mass N inelastically. There is no aggregate uncertainty in this economy and all aggregate variables remain constant.

There is a continuum of firms with measure one in this economy. Each firm i produces heterogeneous intermediate goods with labor as well as equipment and structure using a constant-return-to-scale CES technology

$$Y_{it} = \hat{A}_{it} \left(\alpha_E^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\hat{\alpha}_K} N_{it}^{\hat{\alpha}_N}, \tag{6.1}$$

where parameter α_E (α_S) is the equipment (structure) share, and $\hat{\alpha}_K$ ($\hat{\alpha}_N$) is the capital (labor) share. $\hat{A}_{it}$ represents the firm-specific physical productivity. A final good producer combines goods from individual firms and producing competitively with a CES aggregator:

$$Y_t = \left(\int Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{6.2}$$

where $\sigma \in (1, \infty)$ is the elasticity of substitution between intermediate goods, and the aggregate price index is normalized as one ($P_t = 1$).

Firms' Problem In the beginning of each period, firm i hires workers from the competitive labor market with at a wage, W_t. There are no frictions in the labor market. Subsequently, after completing production, firm i determines investments in equipment and structure for the upcoming period. Investments for equipment and structure are subject to quadratic adjustment costs, given by

$$\Phi\left(K_{it+1}, K_{it}\right) = \frac{\hat{\xi}_K}{2} \left[\frac{K_{it+1}}{K_{it}} - (1-\delta_K) \right]^2 K_{it}, \quad \forall K = \{E\,, S\} \tag{6.3}$$

where δ_K is the depreciation rate, and ξ_K denotes to the intensiveness of adjustment cost.

In addition to adjustment costs, the firm faces capital-specific distortions, denoted as T^E and T^S, when making investment decisions. We solve for optimal labor choice and plug it into the above optimization problem, resulting in

$$\begin{aligned} \mathcal{V}\left(E_{it}, S_{it}, \mathcal{I}_{it}\right) = \max_{E_{i,t+1}, S_{i,t+1}} \mathbb{E}_{it} \Bigg[& G A_{it} \left(\alpha_E^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} \\ & - \sum_{K=\{E,S\}} T_{i,t+1}^K R_K K_{i,t+1} (1-\beta(1-\delta_K)) - \Phi\left(K_{i,t+1}, K_{it}\right) \Bigg] \\ & + \beta \mathbb{E}_{it}\left[\mathcal{V}\left(E_{i,t+1}, S_{i,t+1}, \mathcal{I}_{it+1}\right)\right], \end{aligned} \tag{6.4}$$

where $\alpha_i = \hat{\alpha}_i \left(1 - \frac{1}{\sigma}\right), \forall i = K, N$, $\alpha = \frac{\alpha_K}{1-\alpha_N}$, $A_{it} = \hat{A}_{it}^{\frac{1-\frac{1}{\sigma}}{1-\alpha_N}}$ is the profit productivity, and the constant term $G = (1-\alpha_N) \left(\frac{\alpha_N}{W}\right)^{\frac{\alpha_N}{1-\alpha_N}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_N}}$. R_K for $K = \{E, S\}$ are prices for equipment and structures, and we normalize $R_E = 1$ as numeraire. Moreover, $\mathbb{E}_{it}[\cdot]$ denotes expectations conditional on the firm's information set, $\mathcal{I}_{it}$, which will be discussed in detail below.

Productivity From now on, we will refer to the profit productivity, $a_{it} \equiv \log(A_{it})$ as a firm's productivity. We assume that a_{it} follows to an $\text{AR}(1)$ process,

$$a_{it} = \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \tag{6.5}$$

ρ is the degree of persistence and σ_μ^2 stands for the variance of the innovations μ_{it}.

Investment Wedges The functional forms of $\log(T^E)$ and $\log(T^S)$ are specified as follows

$$\tau_{it}^K = \gamma_K a_{it} + \varepsilon_{it}^K + \chi_i^K, \tag{6.6}$$

in which lowercase letters represent variables after taking the logarithm. The wedges consist of three components: (i) γ_K, the distortions in equipment and structure correlated with a firm's productivity; (ii) ε_{it}^K, the i.i.d. firm–capital-type–specific shocks; and (iii) χ_i^K, the permanent components that capture firm-specific, time-invariant effects.

One of our key deviations from David and Venkateswaran (2019) is that we allow shocks to be correlated across types:

$$\left[\begin{array}{c} \varepsilon_{it}^E \\ \varepsilon_{it}^S \end{array}\right] \sim N\left(0, \left[\begin{array}{cc} \sigma_{\varepsilon^E}^2 & \sigma_{\varepsilon^E\varepsilon^S} \\ \sigma_{\varepsilon^E\varepsilon^S} & \sigma_{\varepsilon^S}^2 \end{array}\right]\right), \text{and} \left[\begin{array}{c} \chi_i^E \\ \chi_i^S \end{array}\right] \sim N\left(0, \left[\begin{array}{cc} \sigma_{\chi^E}^2 & \sigma_{\chi^E\chi^S} \\ \sigma_{\chi^E\chi^S} & \sigma_{\chi^S}^2 \end{array}\right]\right) \tag{6.7}$$

where the $\sigma_{\varepsilon^E\varepsilon^S}$ and $\sigma_{\chi^E\chi^S}$ are covariance of distortions. The non-zero correlations between shocks are consistent with our empirical findings, resulting in a mixed distortion between equipment and structure that would also contribute to the aggregate capital misallocation. Therefore, these two covariance terms are potential candidates for explaining the importance of mixed distortion in aggregate capital misallocation.

Information Firm i does not know its future productivity except for receiving a noisy signal $u_{i,t+1}$

$$u_{i,t+1} = \mu_{i,t+1} + f_{i,t+1}, \quad f_{it+1} \sim \mathcal{N}\left(0, \sigma_f^2\right) \tag{6.8}$$

where the noise shock $f_{i,t+1}$ is normally i.i.d. distributed.

Firm i perfectly observes the transitory shocks and the permanent components of wedges. Thus, the firm's information set is given by $\mathcal{I}_{it} = \left(a_{it}, u_{i,t+1}, \varepsilon_{i,t+1}^E, \varepsilon_{i,t+1}^S, \chi_i^E, \chi_i^S\right)$. We assume that firm i learns using Bayes' rule, and that yields the posterior distribution of productivity, a_{it+1} as following:

$$a_{it+1} \mid \mathcal{I}_{it} \sim N\left(E_{it}\left(a_{it+1}\right), V\right) \tag{6.9}$$

where

$$E_{it}\left(a_{it+1}\right) = \rho a_{it} + \frac{V}{\sigma_f^2} s_{it+1}, \quad V = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_f^2}\right)^{-1} \tag{6.10}$$

6.2 Solving the Model

We solve the model via a perturbation method. Specifically, we log-linearize the firm's Euler equations around their steady states given by $A_{it} = \bar{A}$ and $T_{it}^E = T_{it}^E = 1$, which yields

$$\widetilde{e}_{i,t+1}\left[(1+\beta)\xi_E + 1 - \alpha_E\alpha\right] = \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \widetilde{\tau}_{it+1}^E + \beta\xi_E\mathbb{E}_{it}(\widetilde{e}_{i,t+2}) + \alpha_S\alpha\widetilde{s}_{i,t+1} + \xi_E\widetilde{e}_{i,t} \tag{6.11}$$

$$\widetilde{s}_{i,t+1}\left[(1+\beta)\xi_S + 1 - \alpha_S\alpha\right] = \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \widetilde{\tau}_{it+1}^S + \beta\xi_S\mathbb{E}_{it}(\widetilde{s}_{i,t+2}) + \alpha_E\alpha\widetilde{e}_{i,t+1} + \xi_S\widetilde{s}_{i,t} \tag{6.12}$$

where ξ_E, ξ_S and $\tau_{i,t+1}^E, \tau_{i,t+1}^S$ and rescaled versions of the adjustment cost parameters, $\hat{\xi}_E$, $\hat{\xi}_S$ and the distortion, $\log T_{i,t+1}^E, \log T_{i,t+1}^S$, respectively. We use guess and verify method to solve the two policy functions given below:

$$\widetilde{e}_{i,t+1} = \psi_1^E\widetilde{e}_{it} + \psi_2^E\widetilde{s}_{it} + \psi_3^E\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) + \psi_4^E\varepsilon_{i,t+1}^E + \psi_5^E\varepsilon_{i,t+1}^S + \psi_6^E\chi_i^E + \psi_7^E\chi_i^S \tag{6.13}$$

$$\widetilde{s}_{i,t+1} = \psi_1^S\widetilde{s}_{it} + \psi_2^S\widetilde{e}_{it} + \psi_3^S\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) + \psi_4^S\varepsilon_{i,t+1}^S + \psi_5^S\varepsilon_{i,t+1}^E + \psi_6^S\chi_i^S + \psi_7^S\chi_i^E \tag{6.14}$$

where $\psi_1^E \sim \psi_7^E$ and $\psi_1^S \sim \psi_7^S$ are undetermined coefficients and can be pinned down by Euler equations. In Appendix D, we present the full process of solving our model, a quantitative analysis of the properties of the two policy functions, and a detailed discussion of how different frictions impact future investments in structures and equipment. We also provide insights into how γ influences the estimation results. In the main body of the paper, we focus on the quantitative findings.

7 Quantitative Analysis: Capital-Specific Misallocation Decomposition

7.1 Calibration and Moments

Applying our model to moments from the Compustat, we first calibrate the parameters in preference and production functions following the standard way in the literature. We set the value of discount factor (β) to be 0.95 as the moments are generated by using yearly data. We assume the elasticity of goods substitution (σ) to be 6 in the baseline results, to be consistent with our static framework and also be comparable with the results in David and Venkateswaran (2019).

Firm's production technology is assumed to be constant return to scale. Following Asker et al. (2014), factor shares are calibrated by using the median value of the cross-sectional distribution of factors: capital share in production ($\hat{\alpha}_K$) is set to be 0.33 and equipment share in capital ($\hat{\alpha}_E$) is 0.66. We follow Hulten and Wykoff (1980) to set the

Table 4: Calibration and Estimation

Parameter	Description	Target/value
Preferences/production		
σ	Elasticity of substitution	6
β	Discount rate	0.95
δ_E (δ_S)	Equipment (Structure) Depreciation	0.14 (0.03)
$\hat{\alpha}_K$	Capital share	0.33
$\hat{\alpha}_N$	Labor share	0.67
$\hat{\alpha}_E$	Equipment share	0.66
$\hat{\alpha}_S$	Structure share	0.34
γ	Equipment and structures substitutability	0.3, 1, 4
Productivity		
ρ	Persistence of productivity	$\rho_{a,a_{-1}}$
σ_μ^2	Shocks to productivity	σ_a^2
Frictions		
V	Signal precision	$\rho_{e,a-1}$, $\rho_{s,a_{-1}}$
ξ_E, ξ_S	Equipment/Structure adjustment costs	$\rho_{e,e_{-1}}$, $\rho_{s,s_{-1}}$
γ^E, γ^S	Equipment/Structure Correlated factors	$\rho_{arpe,a}$, $\rho_{arps,a}$
$\sigma^2_{\varepsilon^E}, \sigma^2_{\varepsilon^S}, \sigma^2_{\varepsilon^E\varepsilon^S}$	Transitory factors	σ_e^2, σ_s^2, $\sigma^2_{e,s}$
$\sigma^2_{\chi^E}, \sigma^2_{\chi^S}, \sigma^2_{\chi^S\chi^E}$	Permanent factors	σ^2_{arpe}, σ^2_{arps}, $\sigma^2_{arpe,arps}$

depreciation rates for equipment and structure (δ_E and δ_S) to be 0.14 and 0.03, respectively.

We set the elasticity of substitution between equipment and structures (γ) to be 0.3, the estimated value in Section 4. To tease out the intuition of how imperfect substitutability changes our understanding of the sources of investment frictions, we also experiment with two alternatives: $\gamma = 1$, and $\gamma = 4$. The first panel of Table 4 summarizes our calibration.

Then, we parameterize the law of motion of productivity (a_{it}). We measure firm-level productivity as the model-specific linearized Solow residual $a_{it} = p_{it} + y_{it} - \alpha(\alpha_E e_{it} - \alpha_S s_{it})$, up to an additive constant. We then use the $AR(1)$ specification to estimate the auto-correlation coefficient and variance of residuals (ρ, σ_μ^2) meanwhile controlling for year by industry fixed effects.

Finally, we use SMM to estimate the remaining parameters in the model. In selecting moments, we closely follow David and Venkateswaran (2019), choosing similar moments but for equipment and structures separately. Specifically, we demonstrate in Appendix D that the parameters of interest, $\left\{\xi_K, \mathbb{V}, \gamma_K, \sigma^2_{\varepsilon^K}, \sigma^2_{\chi K}\right\}_{\forall K=E,S}$, can be uniquely identified by the following moments: (1) the autocorrelation of equipment (structure) investment growth ($\rho_{\Delta k, \Delta k_{-1}}$); (2) the correlation between equipment (structure) investment and previous fundamentals ($\rho_{k,a_{-1}}$); (3) the correlation between the marginal product of equipment and structures and current fundamentals ($\rho_{\Delta arpk,a}$); (4) the variance-covariance matrix of equipment and structure growth rates ($\sigma^2_{\Delta k}$); and (5) the variance-covariance matrix of the average product of equipment and structures (σ^2_{arpk}).[35] In aggregate capital level, these moments are also commonly used in the literature to estimate souces of misalloca-

[35] The reason we use the average revenue product of capital (ARPK) rather than the marginal revenue product of capital (MRPK) is twofold. First, directly inferring MRPK from the dataset requires assumptions about the model's steady states and expenditure shares. Second, the moments of ARPK/ARPS remain constant when the value of γ changes, making it more general in the estimation procedure.

tion.[36]

Table 5: Target Moments and their Relevant Channels from Compustat

	ρ	σ^2_μ	$\rho_{\Delta e,a_{-1}}$	$\rho_{\Delta s,a_{-1}}$	$\rho_{\Delta e,\Delta e_{-1}}$	$\rho_{\Delta s,\Delta s_{-1}}$	$\rho_{arpe,a}$	$\rho_{arps,a}$
United States	0.95	0.06	0.09	0.11	-0.34	-0.33	0.51	0.31
	$\sigma^2_{\Delta e}$	$\sigma^2_{\Delta s}$	$\sigma^2_{\Delta e,\Delta s}$	σ^2_{arpe}	σ^2_{arps}	$\sigma^2_{arpe,arps}$		
United States	0.05	0.10	0.02	0.42	0.75	0.24		

Note: The data source of calibration is the Compustat North American (1985 – 2019). When calibrating the productiocity process, the production function is assumed to be Cobb-Douglas where $\gamma = 1$. Average revenue producte of equipment and structures (apre and aprs) are defined as the ratio of sales over equipment or structures.

Calibrated moments from US Compustat are shown in Table 5 which provides us hints for estimation results. For example, the correlation between ARPE and productivity is higher than that between ARPS and productivity, indicating that equipment might be more costly to adjust. Furthermore, the variance of structure investment growth rate is higher than that of equipment investment growth rate. Additionally, the variance of ARPE is smaller than the variance of ARPS. This evidence suggests that the equipment residual distortion might be less volatile in both i.i.d. and permanent shocks.

7.2 Results of Estimation: What Contributes to the Additional Misallocation?

The model matches the moments closely for all values of γ, as reported in Table E.5 of Appendix D. We present the baseline estimation results of parameters in Table 6. In Figure 5, we cluster the frictions into same types and then compute the contributions of each individual type to the loss of welfare (using Lemma 3) by isolating each individual type in the absence of the others. This allows us to measure the TFP impact of each channel relative to the efficient allocation with equalized marginal products across firms.[37]

Adjustment Costs. The estimation results with Compustat show that the equipment adjustment cost increases with γ, while structures adjustment cost decreases with it. To our main specification ($\gamma = 0.3$), the estimates of 1.06 and 1.92 indicates values of 0.20 and 0.16 for $\hat{\xi}_E$ and $\hat{\xi}_S$ in the adjustment cost function. This suggests that in the US, the equipment adjustment cost is roughly 1.25 times greater than the structure adjustment cost, similar to the results in Israelsen (2010).[38] These two numbers are roughly in the midel compared to the capital adjustment costs literature (David and Venkateswaran (2019), Cooper and Haltiwanger (2006), and Bloom (2009)).

[36] For example, Cooper and Haltiwanger (2006) uses the autocorrelation of equipment (structure) investment growth to determine their adjustment costs; Klenow and Willis (2007) uses the correlation between equipment (structure) investment and previous fundamentals to estimate the imperfect information; Bartelsman, Haltiwanger, and Scarpetta (2013), Hsieh and Klenow (2014), and later Bento and Restuccia (2017) use the correlation between the marginal product of equipment and structures and current fundamentals to identify correlated factors.

[37] Since frictions can be correlated with each other, the individually generated misallocation need not sum up to be exactly the total TFP loss we observed from the data.

[38] The result is different from the assumptions from Jermann (2010) and Tuzel (2010).

Table 6: Baseline Estimation Results (Compustat)

Parameters	Description	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 4$
ξ_E	Equipment adjustment cost	1.06	1.68	2.32
ξ_S	Structure adjustment cost	1.92	1.33	1.34
$\mathbb{V}$	Signal precision	0.03	0.03	0.03
γ_E	Equipment correlated factor	-0.34	-0.23	-0.19
γ_S	Structure correlated factor	-0.00	-0.16	-0.21
$\sigma^2_{\varepsilon_E}$	Equipment transitory	0.11	0.09	0.12
$\sigma^2_{\varepsilon_S}$	Structure transitory	0.32	0.08	0.05
$cov(\varepsilon_E, \varepsilon_S)$	i.i.d. correlation factor	-0.19	-0.08	-0.05
$\sigma^2_{\chi_E}$	Equipment permanent factor	1.09	0.29	0.26
$\sigma^2_{\chi_S}$	Structure permanent factor	3.47	0.65	0.28
$cov(\chi_E, \chi_S)$	correlation factor	-1.36	0.15	0.25

Intuitively, the underlying reason behind this pattern could be that plants and machinery are designed and produced for specific purposes. For instance, cash machines are intended solely for shopping. Consequently, when firms need to buy or sell such equipment (cash machines) for further investment or divestment, they encounter lower liquidity in the secondary market. However, structures, such as land, can be easily bought and sold, then repurposed to serve entirely different functions for different firms and operations.

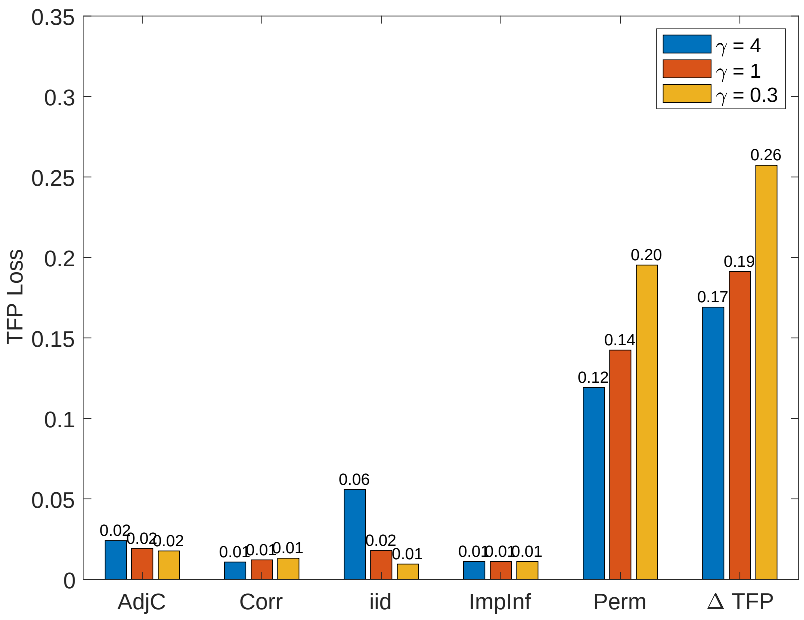
By turning on equipment and structures adjustment cost only, our results show that regardless of the value of γ, the aggregate TFP loss induced by adjustment costs always is always around 2%. This pattern implies that the larger capital misallocation, caused by assuming a smaller number of substitutability γ in measurements, can not be attributed to the change of estimated adjustment costs and productivity dispersion. Hence, compared to a dynamic firm investment "undistorted" benchmark (Asker et al. (2014)), capital hetereogeneity still measures more misallocation.

Imperfect Information. The estimated imperfect information is significantly different from 0 and remains stable as γ decreases from 4 to 0.3. Our estimates of $\mathbb{V} = 0.03$ imply a signal-to-noise ratio of around 1 and US firms could reduce uncertainty by 44% through learning from productivity news. At the aggregate level, imperfect information always causes a 1% TFP loss, aligning qualitatively close to David et al. (2016). This loss is only slightly smaller as γ decreases. Hence, we argue that similar to adjustment costs, increased misallocation due to capital heterogeneity isn't driven by greater inferred information frictions. [39]

Residual Distortions. All three estimated components of investment distortions significantly vary with different values of γ. We estimate the correlations between equipment

[39] It is also worth mentioning that we limit the role of imperfect information at the firm-level. In fact, firms' expectation errors in the aggregate economy can also induce capital misallocation, as evidenced in Ropele et al. (2023) and Wang (2024). As a result, it could be the case that information friction plays a more significant role in explaining misallocation.

Figure 5: Contributions of Frictions on Aggregate Productivity (Compustat)



Note: This table presents the counterfactual total factor productivity (TFP) loss generated when only one type of friction exists in the economy, with varying values of γ. Specifically, **AdjC** refers to costs associated with equipment and structures, **Corr** denotes correlated factors affecting equipment and structures, **iid** indicates transitory shocks to equipment and structures, **ImpInf** stands for imperfect information, and **Perm** refers to permanent factors.

and structure distortions and productivity at 0.35 and 0.002, respectively. Compared to the literature, the equipment correlation is higher than Hsieh and Klenow (2014)'s estimate of 0.09 in the U.S. and falls within the range of Bento and Restuccia (2017) (from 0.22 to 0.74 across different countries). The structure correlation is minimal because land and buildings are typically non-tradable across locations and funded by long-term bonds, making them less liquid than equipment. Overall, correlated factors account for only 1% of the aggregate TFP loss, and their contribution to total misallocation decreases as different types of capital become more complementary.

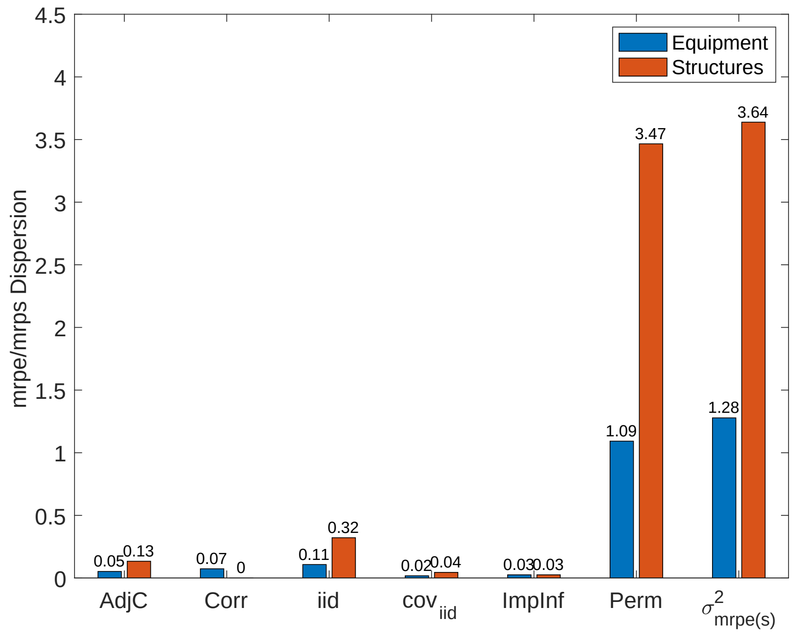
Next, the estimated variances of equipment and structures transitory factors are larger with smaller γ, while their covariance becomes more negative. The increasing variances lead to more misallocation losses while their declining covariance implies less. Jointly, as γ decreases, the transitory components actually induce only a 1% TFP loss, which is unable to account for the extra misallocation observed in the measurement.

Finally, our results show that permanent distortions capture all other sources of misallocation that cannot be explained by the channels mentioned above. The estimated permanent distortions are much more dispersed and negatively correlated with smaller γ, similar to the transitory components. However, the permanent components can generate more than 20% of TFP losses when $\gamma = 0.3$, in stark contrast to the 12% when $\gamma = 4$ as well as the 13% estimate in David and Venkateswaran (2019). Hence, the estimation suggests that firm-specific permanent factors are responsible for the large difference in misallocation measurement when accounting for realistic complementarity. [40]

7.3 Which Sources Make Structures More Misallocated?

Both the measured dispersion of MRPS and MRPE from Section 5.4, as well as the variances of ARPS and ARPE from Table 5, reveal that structures are more misallocated than equipment. To determine whether technological and informational frictions drive these efficiency differences, we compute the MRPE and MRPS dispersion that each capital-type-specific friction can generate by activating each channel while shutting down the others. When we measure each channel's contribution to misallocation, we keep the elasticity γ as 0.3. Figure 6 shows the results.

Figure 6: Comparing of E/S Frictions Contribution



Note: This table presents the counterfactual total factor productivity (TFP) loss generated by each capital type-specific distortion/friction. **AdjC** refers to costs associated with equipment and structures, **Corr** denotes correlated factors affecting equipment and structures, **iid** indicates transitory shocks to equipment and structures, **ImpInf** stands for imperfect information, and **Perm** refers to permanent factors.

[40] What observations in the data support this conclusion? The dispersion moments in MRPE and MRPS from the data are significant, theoretically attributable to adjustment costs, information friction, correlated factors, and two different types of shocks. However, the correlation between equipment/structure investment growth and past period firm fundamentals is small, suggesting that information plays a limited role in explaining misallocation. Furthermore, the variance-covariance matrix of the idiosyncratic shock is also small, limiting the role of the iid shocks. Additionally, the autocorrelation of the two assets is not substantial, indicating that adjustment costs can explain some but not all of the misallocation. Moreover, there is no high correlation between MRPE/MRPS and current firm-level fundamentals, suggesting that the two correlated factors are not the main contributors. Therefore, the permanent shocks account for the majority of the remaining misallocation.

Technological/Informational Frictions. From Figure 6, the equipment adjustment cost generates 0.05 of MRPE dispersion, while the structures adjustment cost generates 0.13 of MRPS dispersion. However, the difference between these two values is small compared to the variances of MRPES and MRPS measured directly from the data. For informational frictions, the dispersion is roughly the same for both MRPE and MRPS, at approximately 0.03. Therefore, neither adjustment costs nor informational frictions significantly increase the dispersion of MRPS compared to MRPE.

Residual Distortions. Focusing on the various components of the two residual distortions, we find that the equipment correlated factor generates 0.07 of MRPE dispersion, while the structures correlated factor has minimal impact. The structures transitory shocks produce 0.32 of MRPS dispersion, nearly three times the 0.11 generated by equipment transitory shocks. However, this difference remains insignificant compared to our observations from the data. In this analysis, two permanent factors are particularly influential. The structures permanent factor accounts for 3.47 of MRPS dispersion, representing 95% of the total MRPS dispersion observed. In contrast, the equipment permanent factor generates 1.09 of MRPE dispersion, which is 85% of the measured MRPE dispersion. Notably, the difference in dispersion generated by these permanent factors aligns closely with the differences measured directly from the data.

To summarize, throughout the quantitative exercises, we find that both adjustment costs and imperfect information play only modest roles in explaining the additional misallocation observed with smaller elasticity γ, as well as the efficiency differences between equipment and structures. Given the significant impact of the two residual distortions, we will further explore several potential economic forces that were not directly modeled in our baseline model in the next section.

8 Candidates in the Residual Distortions

Following the misallocation literature, we primarily consider four candidates behind the residual distortions: financial friction, taxation, heterogeneous technology, and measurement errors. We abstract away from markups (Peters (2020)), as their contribution to aggregate misallocation does not depend on γ when assuming markups arise from intermediate inputs, and their effect is common to both equipment and structures.

8.1 Heterogeneous Financial Frictions

In our baseline model, we abstract away from financial frictions, as the literature has shown that they have only a modest impact on aggregate misallocation (e.g., Midrigan and Xu (2014)). However, heterogeneous financial frictions may help explain the efficiency differences between structures and equipment, as researchers have documented that structures, such as real estate, face relatively small collateral constraints compared to equipment like machines (e.g., Ai, Li, Li, and Schlag (2020), Chaney, Sraer, and Thesmar (2012), and Jermann (2010)).

We extend our model with heterogeneous financial friction by assuming that firms need costly liquidity assets in order to operate production. Specifically, we assume a liquidity cost

$$Y\left(E_{it+1}, S_{it+1}, B_{it+1}\right) = \hat{v} E_{i,t}^{\omega_e} S_{i,t}^{\omega_s} \left(\frac{B_{i,t}}{S_{it} + E_{it}}\right)^{\omega_b} R B_{i,t} \tag{8.1}$$

where B_{it} refers to bond and R is the risk-free rate. The ratio of bonds to the sum of equipment and structures denotes a firm's leverage ratio. We assume $\omega_b > 0$, based on the intuition that more leveraged firms face higher liquidity costs, conditional on having the same stock of equipment and structures. The continuous functional form of this liquidity cost allows us to still solve the model in a linear fashion. In the model equipped with this financial friction, firms' equipment and structures investment curvature will change.

The optimally condition (shown in Appendix F) for the bond demand yields the following structural regression that we can directly test from the dataset

$$\log\left(\frac{B_{it}}{S_{it} + E_{it}}\right) = -\frac{\omega_e}{\omega_b} \log E_{it} - \frac{\omega_s}{\omega_b} \log S_{it} + \text{ Constants} \tag{8.2}$$

where if we bring to the data, we regress the book values of equipment and structures on firms' ratio of bond over total fixed assets.

Table 7: Equip. & Struct. and Debt Leverage Ratio

	$\log \frac{\text{Debt}}{\text{Capital}}$	$\log \frac{\text{Debt}}{\text{Capital}}$	$\log \frac{\text{Debt}}{\text{Capital}}$	$\log \frac{\text{Debt}}{\text{Sales}}$	$\log \frac{\text{Debt}}{\text{Sales}}$
$\log(\text{Structures}_{it}^{\text{Book Value}})$	0.0360***	0.0890***		0.1122***	0.1950***
	(0.0127)	(0.0177)		(0.0140)	(0.0174)
$\log(\text{Equipment}_{it}^{\text{Book Value}})$	-0.0056	-0.0314		-0.0007	0.0543**
	(0.0142)	(0.0243)		(0.0156)	(0.0236)
$\Delta \log(\text{Structures}_{it}^{\text{Book Value}})$			0.0874***		
			(0.0182)		
$\Delta \log(\text{Equipment}_{it}^{\text{Book Value}})$			0.0078		
			(0.0249)		
Firm FE	No	Yes	Yes	No	Yes
Sector-Year FE	Yes	Yes	Yes	Yes	Yes
Observations	59,213	58,065	47,308	59,216	58,065
R^2	0.210	0.677	0.239	0.279	0.703

Table 7 shows the results using Compustat data. In the first three columns, we use the ratio of debt to total fixed assets as the dependent variable, and we also use the ratio of debt to sales as a robustness check. The results indicate that firms holding more structures are more likely to be financially leveraged. On the other hand, we do not find any correlation between equipment holdings and the leverage ratio. The results are similar when using the ratio of debt to sales.

Bringing the reduced-form evidence into our structural model, we use the estimated $\frac{\hat{\omega_e}}{\omega_b}$ and $\frac{\hat{\omega_s}}{\omega_b}$ to discipline the model. We consider these two moments to be most informative about ω_e and ω_s. To infer the remaining two parameters, $\hat{\nu}$ and ω_b, we use two alternative moments from the production data: the variance of MRPE and MRPS growth (first order difference).

Table 8: Estimation Results with Financial Friction

ξ_E	ξ_S	γ_E	γ_S
0.2360	1.4069	-0.7689	-0.2891
$\sigma^2_{\epsilon_E}$	$\sigma^2_{\epsilon_S}$	cov_ϵ	V
0.0236	0.2292	-0.0735	0.0256
ν	ω_b	ω_e	ω_s
0.0320	0.0197	0.0006	-0.0017

Table 8 shows the estimation results. Consistent with our empirical results, even though small, more structures holding will slightly decrease the liquidity costs while equipment has the opposite and more modest effect. When comparing the generated MRPE and MRPS, we find that the generated MRPS is larger with financial friction (4.08 vs. 3.64), while generated MRPE becomes smaller (1.28 vs. 1.07). However, the financial friction will only change the aggregate TFP loss with a small scale (around 1%).

8.2 Tax: "Bonus" Depreciation

Tax policies could be a potential source of capital misallocation (Restuccia and Rogerson (2017)). House and Shapiro (2008) and Zwick and Mahon (2017) have studied the "bonus" depreciation policy, which allows firms to accelerate the schedule for deducting the cost of investment purchases from taxable income. Specifically, let z_N denote the stream of future depreciation deductions owed for investment at sector N

$$z_N = \sum_{t=0}^{T} \frac{1}{(1+r)^t} D_t \tag{8.3}$$

where D_t represent the allowable deduction per dollar of investment in period t, T denote the asset's class life, and r be the risk-adjusted discount rate used by the firm. The variable z_N captures the present discounted value of the pre-tax investment deductions for each dollar invested.

Bonus depreciation enables a firm to deduct a bonus amount, denoted as θ_t, for each dollar invested at the time of the investment. The remaining portion, $1-\theta_t$, is then depreciated following the standard schedule

$$z_{Nt} = \theta_t + (1-\theta_t) z_N \tag{8.4}$$

which provides us the both the cross-sectional and time-series variation. The "bonus" depreciation excludes most of the structures investment, which helps us to study how a single tax policy could potentially lead to difference in MRPE and MRPS dispersion.

We merge the "bonus" depreciation policy with the Compustat, and run the following regression

$$\forall k = e \text{ and } s : \log(mrpk_{it}) = \beta_0^k + \beta_3^k \log(z_{Nt}) \cdot \alpha_i^k + \underbrace{\delta_i + \delta_N + \delta_t + \delta_N \cdot \delta_t}_{FEs} + \varepsilon_{it}; \quad (8.5)$$

where α_i^k is the firm-specific equipment and structures shares. The estimated $\hat{\beta}_3^k$ can be interpreted as: a positive β_3^k means an increase in $\log(z_{Nt})$ leads to a greater increase in $\log(\text{mrpk}_{it})$ for firms with larger α_i^k. From the results, we estimate a $\hat{\beta}_3^e = -0.049^{**}$ and a $\hat{\beta}_3^s = 0.176^{***}$, indicating that this tax policy will be likely to make equipment less misallocated than structures in terms of magnitude. In order to better map the tax-induced misallocation into welfare loss, we run the following regression

$$var(arpk)_{Nt} = \beta_0 + \beta_1 \cdot z_{Nt} + \delta_N + \delta_t + \varepsilon_{Nt} \quad (8.6)$$

which allows us to directly identify the amount of ARPE and ARPS variances correlated with the policy.

Table 9: ARPE & ARPS Variance Induced by "Bonus" Depreciation Policy

	var(arpe)	var(arps)
$\log(z_{Nt})$	18.735	42.577**
	(11.234)	(14.953)
Observations	320	320
R^2	0.767	0.634
Sector FE	Yes	Yes
Year FE	Yes	Yes

Note: the standard errors are clustered at the sector level.

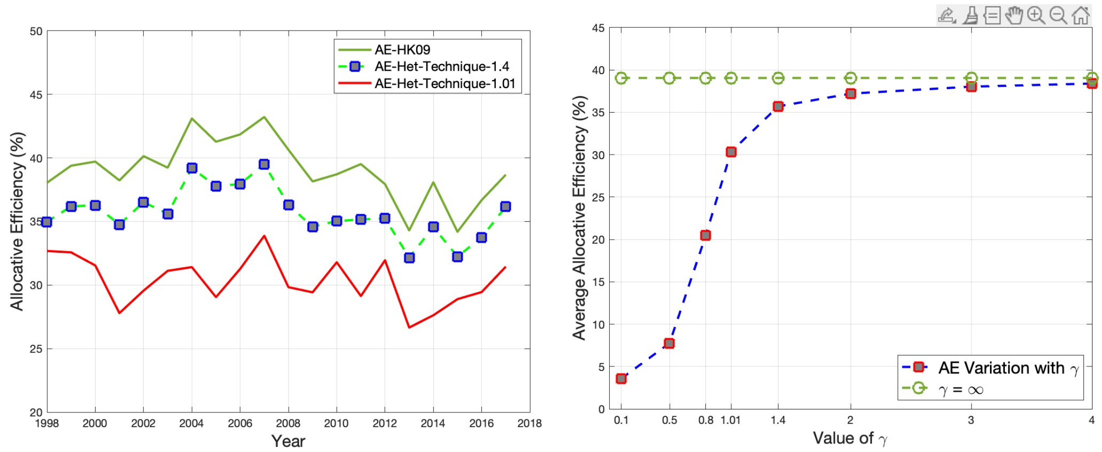
Table 9 shows the results with sectors whose the number of firms is more than 40. We can see that the results do not suggest that the "Bonus" depreciation policy has a significant impact on the ARPE dispersion. However, it does increase the ARPS dispersion.

8.3 Contribution of Heterogeneous Techniques to Misallocation

In both of our static framework and dynamic model, we assume that firms in the same sector use the same production technology. A potential concern of disaggregating capital will be the heterogeneity in equipment/structures elasticity. Even though not perfect, we use different methods in both static and dynamic framework to measure the contribution of equipment/structures heterogeneity on misallocation.

Extensive Margin: Heterogeneous Techniques. In static framework, we group firms that use different types of capital together in the same sector, then allow firms in different groups within the same sector have different production technology.[41] For example, under the manufacturing sector, one group of firms contains firms using only machinery and building in their production, while another group contains firms that only use machinery, building, and transportation.[42]

Figure 7: Measured AE with Heterogeneous Technoques



Note: The above figures show the measured allocative efficiency with heterogeneous techniques. The first panel provides the time series evidence for AE, while the second panel shows how AE changes with different values of the elasticity of capital substitution.

The left figure in Figures 7 shows the results of our measurement with $\gamma = 1.4$. We still consistently measure a difference between homogeneous and heterogeneous capital from 5.30% to 12.23% depending on different years. Moreover, in the right hand side of Figure 7, we can see that when Convention 1 does not hold (due to heterogeneous technology with a sector), measured allocative efficiency still increases with the value of γ. This shows the robustness of our empirical measurement results and the claim of underestimation of misallocation costs in conventional models.

Intensive Margin: Estimating Heterogeneous Input Elasticities. We follow Salgado, Ozkan, Hubmer, Hong, and Chan (2024) to directly identify the heterogeneous production elasticity at the firm-year level using the Gandhi et al. (2020) estimator (henceforth GNR). We assume that equipment (e_{it}) and structures (s_{it}) are dynamic inputs in production, while cost-of-goods-sold is the flexible input.

[41] This actually also solves some caveats of our empirical measurements. Firstly, there might be sample selection bias. As we disaggregate the total fixed asset into more and more different types of assets, the number of surviving observations—those with non-zero reported usage on all types of assets—declines. We have to drop the firms that have missing values or do not use all of the assets; otherwise, the measured firm-level productivity will be biased. Eventually, when we measure using six assets, almost sixty percent of the observations in the sample are dropped. More importantly, we found that mostly large firms tend to use all six types of assets, so it is very likely that we are only measuring the cost of misallocation among large firms. Finally, since different firms use different types of assets, a harmonized method of measurement is in need to compute allocative efficiency.

[42] It is worth noting that in this setting, our Convention 1 will no longer hold, since firms have different production functions in the same sector. We solve the static model to measure the efficient capital allocation in this economy: by fixing the supply of all assets and labor, we guess the prices iteratively until all factor markets clear.

The production function estimation methodology developed by GNR offers several advantages that are particularly attractive for our problem. First, the elasticity estimates are robust to potential adjustment costs in the investment process, as demonstrated by GNR's Monte Carlo exercises. Second, it offers a consistent estimation of the input elasticities as long as the evolution of firm-specific input prices follows a time-varying Markov process, as shown by Luparello (2023), which nests the stochastic processes for input distortions in our model. Lastly, the GNR estimator yields a heterogeneous production elasticity at the firm-year level. Not only does it recover the average elasticity, but it also provides accurate estimates for the dispersion of input elasticities across firms, the key object of our interest.

Applying the GNR method, we recover substantial within-sector-year heterogeneity in the (log) of equipment and structure input elasticities, with $\text{Var}(\widetilde{\alpha}_{Eit}) = 5.28$ and $\text{Var}(\widetilde{\alpha}_{Sit}) = 13.35$. However, the large measured technology dispersion does not mean that our baseline measurements overstate the measured misallocation. In fact, the dispersion of $\widetilde{\alpha}_{Eit}$ and $\widetilde{\alpha}_{Sit}$ explains less than 2% of the variation in the $mrpe_{it}$ and $mrps_{it}$ computed with our baseline model. After adjusting for the estimated heterogeneous input elasticities, the measured σ^2_{mrpe} and σ^2_{mrps} are 2.20 and 7.97, respectively, which are almost twice as large as our baseline estimates. The correlated distortion $\sigma_{mrpe,mrps}$ is now measured to be more negative, at -2.69. Together, our results suggest that accounting for the heterogeneity in production technology would result in an even larger measured misallocation loss than our baseline.

Intensive Margin: Heterogeneous Factor-Augmenting Productivity As an alternative method, we follow one way prposed in David and Venkateswaran (2019) in order to identify the effect of heterogeneous technology. The details can be found in Appendix G and we sketch the main processes here. First of all, we can assume that $\tau^E_{it} = \tau^S_{it} = \tau^N_{it}$, which in the first order implies

$$arpe_{it} - arpn_{it} = \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}}\left(1-\frac{1}{\alpha_E}\right)^{\frac{1}{\gamma}-1}\frac{1}{\alpha_E}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{\frac{1}{\gamma}}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\kappa}_{it} + \text{constant} \tag{8.7}$$

or

$$arps_{it} - arpn_{it} = \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{\frac{1}{\gamma}-1}\frac{1}{\alpha_E}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{-\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{-\frac{1}{\gamma}}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\kappa}_{it} + \text{constant} \tag{8.8}$$

where $\widetilde{\kappa}_{it}$ is firm-level equipment/structures ratio and κ is its deterministic steady state. $arpe_{it}$, $arps_{it}$ and $\widetilde{\kappa}_{it}$ can all be computed from data so that we can calculate the variance of $\widetilde{\alpha}_{Eit}$. Applying this method in US Compustat data, we compute variances of $\widetilde{\alpha}_{Eit}$, $\widetilde{\alpha}_{Sit}$ and their covariance. We found that the loss of misallocation which heterogeneous elasticity can generate is 0.018%, which is tiny compared to the number of the underestimation.

8.4 Measurement Errors

Measurement error is a common concern about measuring capital misallocation. Take it to our case, the concern is that will the measurement error be larger when using the more detailed, disaggregating capital data. We follow the empirical strategy developed in Bils et al. (2021) to estimate the role of additive measurement error in our empirical results. Specificallly, we run the following regression

$$\Delta py_{it} = \beta_0 + \beta_1 \cdot tfpr_{it} + \beta_2 \cdot \Delta I_{it} - \beta_2(1-\beta_3)tfpr_{it} \cdot \Delta I_{it} + D_{jt} + \varepsilon_{it}, \tag{8.9}$$

in which Δpy_{it} and ΔI_{it} are firm revenue and inputs changes. The independent variable $tfpr_{it}$ is refereed to firm level revenue TFP (in logarithm) following the definition in equation (3.1). D_{jt} captures industry-year fixed effects. The basic idea behind this identification is that conditional on the same TFPR, the relative growth rate of firm revenue to input can be used to identify the addictive measurement error, under certain assumptions [43]. In equation (8.9), β_3 means the ratio of the true dispersion of TFPR to its measuement counterpart (with measurement errors).

We try two different specification to test if we encounter greater measurement errors with using more disaggregated data. We first use each individual type of capital (e.g. equipment or structure) as the input ΔI_{it}, and compared the estimated β_3 when using total fixed assets as the input as in David and Venkateswaran (2019). This exercise will tell whether different types of capital themselves have more measurement errors. In the second specification, we combine different types of capital and labor as the input bundle, and test whether it has more measurement errors than the input bundles in Bils et al. (2021).

The results from our first specification show that in the US, using the equipment or structures alone to measure capital misallocation do not suffer more measurement errors compared to using total fixed asset. Using the Compustat data, we estimated a $\beta_3^e = 0.11$ for equipment, $\beta_3^s = 0.07$ for structures and $\beta_3^k = 0.12$. The results from our second specification show that $\beta_3^{\text{es bundle}} = 0.11$ when $\gamma = 0.3$, not significantly different from using the total fixed assets. As a concern raised in Bils et al. (2021) that non Cobb-Douglas production function might bias the result, we also try to measure it by assuming $\gamma = 1$ and the results are similar.

8.5 Robustness: Estimating with Varied Moments

We show the estimation of adjustment costs, imperfect information and investment wedges cause loss of aggregate TFP with changed moments in Table 10. Specifically, we

[43] In order to use this empirical specification, we follow the assumptions in Bils et al. (2021). Except for that measurement errors are additive, we need to assume that their role in a plant's TFPR is orthogonal to its true wedge distortions. Moreover, we assume the addictive measurement errors affect all inputs the same. Finally, we also need to assume that productivity, distortion wedges and measurement errors are i.i.d., and they are evaluated in first-order approximation.

re-measure firm-level productivity for each value of γ, and generate moments accordingly, and estimate the model with each γ and moments pairs.

Table 10: Contributions of Frictions on Aggregate Productivity (US) with Changed Moments

Elasticity of Substitution	ξ_{E+S}	$\mathbb{V}$	τ_{E+S} γ_{E+S}	ε_{E+S}	χ_{E+S}	Δa
$\gamma = 4$	3%	0%	10%	1%	9%	14%
$\gamma = 1$	2%	1%	1%	2%	14%	19%
$\gamma = 0.3$	3%	0%	9%	1%	16%	21%

From the results, the contribution of adjustment costs and imperfect information are still quite stable with different numbers of γ and moments. The correlated and transitory factors might vary a lot across different specification, however, we still estimate variances of permanent factors steadily increase with smaller number of γ. This indicates that changing moments accordingly will not change our main results.

8.6 Robustness: Estimation Using India ASI Data

We also apply India ASI data to our model to examine if adjustment costs and imperfect information can explain any extra misallocation measured by finite elasticity of equipment and structures substitution. We follow the main procedure in the baseline estimation, keeping the moments fixed and estimate the model with $\gamma = 0.3$, 1 and 4. Results are attached in Table 11.

Table 11: Contributions of Frictions on Aggregate Productivity (India)

Elasticity of Substitution	ξ_{E+S}	$\mathbb{V}$	τ_{E+S} γ_{E+S}	ε_{E+S}	χ_{E+S}	Δa
$\gamma = 4$	4%	1%	2%	0%	34%	41%
$\gamma = 1$	4%	1%	3%	0%	33%	43%
$\gamma = 0.3$	4%	1%	1%	0%	47%	54%

Similar to the results in the US, the extra TFP loss induced by smaller number of γ can not be explained by the adjustment costs and imperfect information: across different γ, adjustment costs and imperfect information constantly contribute 4% and 1% unit of TFP loss. Different from the US, the aggregate TFP loss is significant larger than that of the US.

9 Conclusions

In this paper, we formalize the role of capital heterogeneity in studying capital misallocation. We propose a new framework which captures the heterogeneity and interactions of distortions among different capital types. Formally, we show that ignoring assets heterogeneity leads to an underestimate measure of capital misallocation, ranging from to 2% to 26% of TFP loss in different countries. Finally, to investigate the mechanisms behind heterogeneous asset-specific misallocation and their joint effect on the aggregate economy, we construct and estimate a firm dynamics model with two categories of capital: equipment and structure. Quantitative analysis reveals the importance of equipment-specific adjustment cost and firm-specific permanent distortions.

Although not explicitly discussed, the policy implications of this paper is clear. As different types of assets face distortions of different nature, any asset-neutral policy to incentives the firm's investment might not be optimal. Our analysis entails that policies that are targeted to reduce frictions in the equipment market should be key to improve allocative efficiency. Moreover, as the model only focuses on explaining the average misallocation across years, the increasing trend of capital misallocation is left unexplained. Future work should be directed to explaining the asset-origin of rising misallocation in the U.S. and provide better understanding of the mechanisms at play.

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Appendices

Online Appendix (Not for Publication)

A Measurement Framework with Multiple Types of Capital

A.1 Static Measurement Framework

We solve firm's profit maximization problem with a CES demand in a static environment now. As it's shown in section 2, firm's profit maximization problem is given by

$$\max_{\{P_i,Y_i,K_{mi},L_i\}} (1+\tau_{Yi}) P_i Y_i - \sum_{m=1}^{M} (1+\tau_{Kmi}) R_m K_{mi} - W L_i,$$
$$\text{subject to } : Y_i = Y \left(\frac{P}{P_i}\right)^{\sigma}.$$

The dual cost minimizing problem of firm's profit maximization can be given by

$$C(Y_i) \equiv \min_{\{\{K_{mi}\}_{i=1}^{m=M},L_i\}} \left[\sum_{m=1}^{M} (1+\tau_{Kmi}) R_m K_{mi} + W L_i \middle| A_i \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha} = Y_i \right].$$

By simply assuming that $RK = \sum_{m=1}^{M} (1+\tau_{Kmi}) R_m K_{mi}$ and $K = \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$, we can directly borrow the cost function from Cobb-Douglas and CES production function to solve for the cost minimization problem

$$C_{Y_i}(K_{1i}, K_{2i}, ..., K_{mi}, L_i) = \frac{1}{A_i} \left[\frac{\left[\sum_{m=1}^{M} \alpha_m \left((1+\tau_{K_{mi}})R_m\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}{\alpha} \right]^{\alpha} \cdot \left(\frac{W}{1-\alpha}\right)^{1-\alpha}. \tag{A.1}$$

Firm i chooses its price P_i by exerting its constant markup on the cost

$$P_i = \frac{\sigma}{\sigma-1} \frac{1}{A_i} \left[\frac{\left[\sum_{m=1}^{M} \alpha_m \left((1+\tau_{K_{mi}})R_m\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}{\alpha} \right]^{\alpha} \cdot \left(\frac{W}{1-\alpha}\right)^{1-\alpha}. \tag{A.2}$$

We also know from the CES demand that $P_i Y_i = P Y^{\frac{1}{\sigma}} Y_i^{1-\frac{1}{\sigma}}$, and plug it into the firm's problem

$$\max_{\{P_i,Y_i,K_{mi},L_i\}} (1+\tau_{Yi}) P Y^{\frac{1}{\sigma}} Y_i^{1-\frac{1}{\sigma}} - \sum_{m=1}^{M} (1+\tau_{Kmi}) R_m K_{mi} - W L_i.$$

In a static measurement framework, the aggregate price and output, P and Y, are constants. Hence, the FOC wrt. different types of capital and labor are given by

$$\text{FOCs } [K_{mi} \& K_{ni}] : \frac{\alpha_n}{\alpha_m} \left[\frac{R_m(1+\tau_{K_{mi}})}{R_n(1+\tau_{K_{ni}})} \right]^{\gamma} = \frac{K_{ni}}{K_{mi}} \tag{A.3}$$

To see how the elasticity affects the use of K_{mi}, let the total expenditure in capital $RK = \sum_{m=1}^{M} R_m K_{mi}$ and the capital bundle $K = \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$. The share of K_{mi} on K follows

$$\frac{K_{mi}}{\left(\sum_{n=1}^{M} \alpha_n^{\frac{1}{\gamma}} K_{ni}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}} = \frac{K_{ni}}{\frac{1}{R} \sum_{m=1}^{M} R_m K_m} = \alpha_m \left[\frac{R_m (1+\tau_{Kmi})}{\left(\sum_{n=1}^{M} \alpha_n R_n^{1-\gamma} (1+\tau_{Kni})^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right]^{-\gamma}. \tag{A.4}$$

A.2 Example: Cobb-Douglas Special Case $\gamma = 1$

To better understand how input distortions affect allocative efficiency, we now consider a simple example with $\gamma = 1$. This is a tractable special case where a closed-form aggregate production function exists for a distorted economy, and how distortions might lower aggregate productivity could be made very transparent. In this case, different capital inputs are neither complements nor substitutes. The firm's production function becomes Cobb-Douglas in all its inputs:

$$Y_i = L_i^{1-\alpha} \left(\Pi_{m=1}^{M} K_{mi}^{\alpha_m} \right)^{\alpha} \tag{A.5}$$

In the distorted economy, we have the allocation of m-th type capital inputs satisfy:

$$K_{mi} \propto \frac{1}{(1+\tau_{Kmi})} \left[\frac{A_i}{(1+\tau_{Li})^{(1-\alpha)} \Pi_{n=1}^{M} (1+\tau_{Kni})^{\alpha_n \alpha}} \right]^{\sigma-1}, \tag{A.6}$$

where the quantity of K_{mi} is inversely proportional with the its type-specifc capital wedge, $(1+\tau_{Li})$, and is proportional to the wedge-adjusted TFP. This is in stark contrast of the efficient allocation described in Lemma 1. Here, not only the wedge on m-th capital will lower the firm i's demand on m-th capital, the input wedges of other factors will also proportionally lower i's demand on m-th capital due to the Cobb-Douglas nature of production. By building upon the firm-level equilibrium input choices and factor market clearing conditions, we can easily derive the aggregate production function as the following:

$$Y = TFP \cdot \prod_{m=1}^{M} K_m^{\alpha_m} L^{1-\alpha}, \tag{A.7}$$

$$TFP = \left[\sum_{i=1}^{N} \left(A_i \frac{1}{(1+\tau_{Li})^{1-\alpha} \prod_{m=1}^{M} (1+\tau_{Kmi})^{\alpha_m \alpha}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \tag{A.8}$$

where the aggregate output Y, is determined by the aggregate TFP, along with the total factor supplies. Furthermore, we can see that the aggregate TFP, accounts for the combined effects of technology and the distortions associated with different inputs. Let us focus on the capital distortions here. The component $\prod_{m=1}^{M} (1+\tau_{Kmi})^{\alpha_m \alpha}$ captures the firm-level total effect of distortions related to different capital types. Each capital type is subject to its own distortion. The net effect of these distortions on TFP depends on the relative importance of each capital type (captured by α_m) and the size of the distortion itself. If the distortion (τ_{Kmi}) is significant for a particularly crucial capital type m, it can considerably dampen the TFP, thereby reducing aggregate productivity. Also, it is particularly damaging when the distortion is paired with a firm with higher productivity (A_i) as it would drag the aggregate productivity further away from the efficient frontier.
The allocative efficiency in this economy also takes a closed form:

$$AE = \frac{Y}{Y^e} = \frac{TFP}{TFP^e} = \frac{\left(\sum_{i=1}^{N} \left(\frac{A_i}{(1+\tau_{Li})^{1-\alpha} \prod_{m=1}^{M} (1+\tau_{Kmi})^{\alpha_m \alpha}} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}}{\left(\sum_{i=1}^{N} A_i^{\sigma-1} \right)^{\frac{1}{\sigma-1}}} \tag{A.9}$$

In the Cobb-Douglas context, the allocative efficiency metric assesses the extent to which a sector's actual TFP diverges from its efficiency frontier. Notably, these distortions influence the aggregate TFP, thereby diminishing the overall efficiency with which aggregate inputs are transformed into output.

A.3 Proof of Lemma 1

The proof is of two parts. First of all, we derive the optimal allocation of capital and labor in a static environment, then show the aggregation production function of the economy. We start our first part of proof from a firm's profit maximization problem. Firm i's capital and labor hiring problem is given by the following optimization problem:

$$\max_{\{K_{1i}, K_{2i}, \ldots, K_{Mi}, L_i\}} P_i Y_i - \sum_{m=1}^{M} R_m K_{mi} - W L_i \tag{A.10}$$

where the first order conditions are:

$$\text{FOC }[K_{mi}]: P_i Y_i \cdot \alpha \alpha_m^{\frac{1}{\gamma}} \cdot \frac{K_{mi}^{-\frac{1}{\gamma}}}{\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}} = R_m, \forall m \in \{1, 2, ..., M\} \tag{A.11}$$

$$\text{FOC }[L_i]: P_i \frac{Y_i}{L_i}(1-\alpha) = W \tag{A.12}$$

Combining the FOC w.r.t two different kinds of capital we can achive the following optimal capital ratio equation:

$$\frac{K_{1i}}{K_{mi}} = \left(\frac{R_m}{R_1}\right)^{\gamma} \cdot \frac{\alpha_m}{\alpha_1} = \frac{K_1}{K_m} \tag{A.13}$$

Since in such an environment, the aggregate type specific capital and labor supply are determined beforehand, the optimal capital ratios are always constant. Moreover, we can rewrite firm i's production technology Y_i as:

$$\begin{aligned} Y_i &= A_i \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha} \\ &= A_i \left[\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} \left(\frac{K_{mi}}{K_{1i}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\alpha} K_{1i}^{\alpha} \cdot L_i^{1-\alpha} \\ &= A_i \left[\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} \left(\frac{K_{m}}{K_{1}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\alpha} K_{1i}^{\alpha} \cdot L_i^{1-\alpha} \end{aligned} \tag{A.14}$$

Then, the social planner's problem is give by:

$$\max_{\{\{K_{mi}\}_{m=1}^{M}, L_i\}} Y = \left(\sum_{i=1}^{N} Y_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\text{s.t. } \sum_{i=1}^{N} K_{1i} = K_1, \sum_{i=1}^{N} L_i = L$$

The Lagrangian is followed by:

$$\mathcal{L} = Y + \lambda_1 \left(K_1 - \sum_{i=1}^{N} K_{1i}\right) + \mu \left(L - \sum_{i=1}^{N} L_i\right) \tag{A.15}$$

First order condition w.r.t. K_{1i} and L_i are given by:

$$\begin{aligned} Y^{\frac{1}{\sigma}} Y_i^{1-\frac{1}{\sigma}} \alpha K_{1i}^{-1} &= \lambda_1 \\ Y^{\frac{1}{\sigma}} Y_i^{1-\frac{1}{\sigma}} (1-\alpha) L_i^{-1} &= \mu \end{aligned} \tag{A.16}$$

Combine FOC $[K_{1i}]$ and $[K_{1j}]$:

$$\frac{K_{1i}}{K_{1j}} = \left(\frac{Y_i}{Y_j}\right)^{1-\frac{1}{\sigma}} = \left(\frac{A_i}{A_j}\right)^{(\sigma-1)} \tag{A.17}$$

Finally, it is straightforward to see that $\frac{L_i}{L_j} = \frac{K_{1i}}{K_{1j}}$, and we plug these optimal conditions into our budget constraints:

$$\sum_{i=1}^{N}\left(\frac{A_i}{A_j}\right)^{(\sigma-1)} K_{1j} = K_1 \Rightarrow K_{1j} = \frac{A_j^{\sigma-1}}{\sum_{i=1}^{N} A_i^{\sigma-1}} K_1 \tag{A.18}$$

which completes the first part of the proof of Lemma 1. For the second part, we need derive the aggregate production function that under the optimal capital and labor allocations. To show this, we plug the optimal capital and labor decision in firm i's production function:

$$\begin{aligned} Y_i &= A_i \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha} \\ &= A_i \left[\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} \left(\frac{A_i^{\sigma-1}}{\sum_{j=1}^{N} A_j^{\sigma-1}} K_m\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\alpha} \left(\frac{A_i^{\sigma-1}}{\sum_{i=1}^{N} A_j^{\sigma-1}} L\right)^{1-\alpha} \\ &= \frac{A_i^{\sigma}}{\sum_{j=1}^{N} A_j^{\sigma-1}} \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L^{1-\alpha} \end{aligned} \tag{A.19}$$

The final good producer aggregates all intermediate good to generate the final good by:

$$\begin{aligned} Y &= \left(\sum_{i=1}^{N} Y_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left[\sum_{i=1}^{N} \left(\frac{A_i^{\sigma}}{\sum_{j=1}^{N} A_j^{\sigma-1}} \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ &= \left(\sum_{i=1}^{N} A_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha} \end{aligned} \tag{A.20}$$

where $\left(\sum_{i=1}^{N} A_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ is just the efficient level productivity TFP^e. This completes our second part of proof of Lemma 1.

A.4 Proof of Lemma 2

We aim to prove that the measured firm-level TFP will decrease with the chosen elasticity of capital substitution γ. The set of production technology that we are interested in is as the following form:

$$Y_i = A_i \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha}, \forall M \in \{1, 2, 3, ...\} \tag{A.21}$$

We neglect time t and i here since our framework is purely static and focus on one individual firm. Apparently, we only need to know the sign of:

$$\begin{aligned} \frac{\partial A_i}{\partial \gamma} &= \frac{\partial \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}}{\partial \gamma} \\ &= \frac{\partial \left(\sum_{m=1}^{M-1} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} + \alpha_M \right)^{\frac{\gamma}{\gamma-1}} K_{Mi}}{\partial \gamma} \end{aligned} \tag{A.22}$$

Notice that we can rescale K_{mi} such that $\alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} = \alpha_m \widetilde{K}_{mi}^{\frac{\gamma-1}{\gamma}}$. We set $x = \frac{\gamma-1}{\gamma}$, then compute:

$$\begin{aligned} &\frac{\partial \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right)^{\frac{1}{x}}}{\partial x} = \exp \left[\frac{1}{x} \ln \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right) \right] \\ &\cdot \left[-\frac{1}{x^2} \ln \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right) + \frac{1}{x} \frac{1}{\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M} \left(\sum_{m=1}^{M-1} \alpha_m K_m^x \ln(K_m) \right) \right] \\ &\propto x \cdot \left(\sum_{m=1}^{M-1} \alpha_m K_m^x \ln(K_m) \right) - \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right) \cdot \ln \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right) \\ &= -\sum_{m=1}^{M-1} \alpha_m K_m^x \ln \left(\frac{\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M}{K_m^x} \right) - \alpha_M \ln \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right) \end{aligned} \tag{A.23}$$

Using the fact that $\log(x) \leq x - 1$, we have:

$$\frac{\partial \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M \right)^{\frac{1}{x}}}{\partial x} \geq \sum_{m=1}^{M-1} \left[\alpha_m K_m^x \left(\frac{\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M}{K_m^x} - 1 \right) + \alpha_M \left(\sum_{m=1}^{M-1} \alpha_m K_m^x + \alpha_M - 1 \right) \right] \tag{A.24}$$

If we simplify the right hand side of above equation, we will find that it is just zero. Hence, we have proved that:

$$\frac{\partial A_i(\mathcal{D}, \mathcal{P}, \gamma)}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[\frac{R_i^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_{mi}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} L_i^{1-\alpha}} \right] \leq 0, \ \forall \mathcal{D}, \mathcal{P} \text{ and } i. \tag{A.25}$$

A.5 Proof of Proposition 1

First, we note that the ratio of the measured allocative efficiency for two different elasticities of substitution of capital, $\gamma' > \gamma$, can be written as:

$$\frac{AE(\gamma)}{AE(\gamma')} = \frac{\frac{Y}{Y^e(\gamma)}}{\frac{Y}{Y^e(\gamma')}} = \frac{Y^e(\gamma')}{Y^e(\gamma)}$$

As the measured aggregate output Y is the same for both choices of elasticity of capital substitution (from the same data $\mathcal{D}$) , what matters for the AE measurement in the two different cases are the difference in the efficient counterfactual $Y^e(\gamma)$ and $Y^e(\gamma')$. Using Lemma 2, since the efficient aggregate production function takes the following form,

$$Y^e(\gamma) = TFP^e \cdot \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\cdot\alpha} L^{\alpha},$$

we can derive that:

$$\frac{AE(\gamma)}{AE(\gamma')} = \frac{Y^e(\gamma')}{Y^e(\gamma)} = \underbrace{\frac{\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)}}{\left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)}}}_{\text{Productivity Effect}} \underbrace{\frac{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma'}} K_m^{\frac{\gamma'-1}{\gamma'}}\right)^{\frac{\gamma'}{\gamma'-1}\cdot\alpha}}{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha}}}_{\text{Aggregate Input Effect}}$$

There are two effects on determining the direction of measurement: (1) *productivity effect* and (2) *aggregate input effect*. We now analyze these two effect separately:

Productivity Effect Lemma 1 established that the measured firm-level productivity $A_i(\mathcal{D}, \mathcal{P}, \gamma)$ is decreasing with the choice of elasticity of capital substitution γ, $\frac{\partial A_i(\mathcal{D},\mathcal{P},\gamma)}{\partial \gamma} \leq 0$. Therefore we have that for every firm i, since $\gamma' > \gamma$,

$$A_i(\gamma') < A_i(\gamma).$$

Since each element of the sum in the measured efficient aggregate productivity function $\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)}$ is smaller than $\left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)}$, we must also have the measured aggregate efficient productivity is increasing in the choice of elasticity of capital substitution γ:

$$\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)} < \left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)}, \forall \gamma' > \gamma.$$

Using the fact that $\gamma \in (0, \infty)$ and $\sup_{\gamma} = \infty$, we must have that for any finite γ,

$$\left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)} > \lim_{\gamma\to\infty}\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)},$$

which implies that a researcher that assumes heterogeneous capital must always measure a higher level of efficient aggregate productivity than one assumes homogeneous capital.

Aggregate Input Effect If we do not assume Convention 1, we have that for $\gamma < \gamma'$:

$$\frac{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha}}{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma'}} K_m^{\frac{\gamma'-1}{\gamma'}}\right)^{\frac{\gamma'}{\gamma'-1}\cdot\alpha}} \leq 1.$$

This is obvious since for the given capital quantity K_m, the larger the flexibility in production, the larger the aggregate input bundle would be. However, under our measurement Convention 1 where $\alpha_m = \frac{K_m}{\sum_m K_m}$, we have that the size of the aggregate input bundle does not depend on the assumed elasticity γ:

$$\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha} = \left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma'}} K_m^{\frac{\gamma'-1}{\gamma'}}\right)^{\frac{\gamma'}{\gamma'-1}\cdot\alpha} = \left(\sum_{m=1}^{M} K_m\right)^{\alpha}$$

Therefore, the aggregate input effect is completely muted under Convention 1.
We now summarize our analysis of the two effects,

$$\frac{AE(\gamma)}{AE(\gamma')} = \underbrace{\frac{\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)}}{\left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)}}}_{\text{Productivity Effect}<1} \underbrace{\frac{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma'}} K_m^{\frac{\gamma'-1}{\gamma'}}\right)^{\frac{\gamma'}{\gamma'-1}\cdot\alpha}}{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma'}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha}}}_{\text{Aggregate Input Effect}=1} < 1,$$

and the measured AE will always be larger for the researcher that assumes a higher degree of elasticity of substitution $\gamma' > \gamma$. Taking γ' to infinity, we arrive at

$$AE(\gamma, \mathcal{D}, \mathcal{P}) < \lim_{\gamma\to\infty} AE(\gamma, \mathcal{D}, \mathcal{P}),$$

which concludes the proof of the proposition.
The easiest way to see this is that, pick random γ and γ', sastifying that $\gamma \leq \gamma'$. From

proposition 1 we can see that:

$$\frac{AE(\gamma)}{AE(\gamma')} = \underbrace{\frac{\left(\sum_{i=1}^{N} A_i(\gamma')^{\sigma-1}\right)^{1/(\sigma-1)}}{\left(\sum_{i=1}^{N} A_i(\gamma)^{\sigma-1}\right)^{1/(\sigma-1)}}}_{\text{Productivity Effect}<1} \underbrace{\frac{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma'}} K_m^{\frac{\gamma'-1}{\gamma'}}\right)^{\frac{\gamma'}{\gamma'-1}\cdot\alpha}}{\left(\sum_{m=1}^{M} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma'}}\right)^{\frac{\gamma}{\gamma-1}\cdot\alpha}}}_{\text{Aggregate Wedge Effect}=1} < 1$$

and this completes the proof.

A.6 Proof of Corollary 1

We need to show that finer disaggregation of capital leads to the smaller number of measured allocative efficiency and larger welfare loss. Compare two situations, one with $M-1$ types of capital, $\{K_1, K_2, ..., K_{M-2}, K_{M-1}\}$, and another with disaggregating the $M-1$-type capital into K'_{M-1} and K_M, where $K'_{M-1} + K_M = K_{M-1}$. In the first situation, the researcher is not aware of the fact that $M-1$-th type of capital can be further disaggregated. Nevertheless, in either situation, the researcher can assign the correct weight following convention 1. From a researcher's perspective, their productivities in two different scenarios should be defined as following:

$$A = \frac{R^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{m=1}^{M-2} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}} + \alpha_{M-1}^{\frac{1}{\gamma}} K_{M-1}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L^{1-\alpha}} \tag{A.26}$$

$$A' = \frac{R^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{m=1}^{M-2} \alpha_m^{\frac{1}{\gamma}} K_m^{\frac{\gamma-1}{\gamma}} + \alpha'^{\frac{1}{\gamma}}_{M-1} K'^{\frac{\gamma-1}{\gamma}}_{M-1} + \alpha_M^{\frac{1}{\gamma}} K_M^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha} L^{1-\alpha}} \tag{A.27}$$

To tell the difference between A and A', we only need to compare $\alpha_{M-1}^{\frac{1}{\gamma}} K_{M-1}^{\frac{\gamma-1}{\gamma}}$ and $\alpha'^{\frac{1}{\gamma}}_{M-1} K'^{\frac{\gamma-1}{\gamma}}_{M-1} + \alpha_M^{\frac{1}{\gamma}} K_M^{\frac{\gamma-1}{\gamma}}$. We assume $\gamma > 1$ at this moment, but the proof still holds otherwise. When the researcher fails to recognize the fact that K_{M-1} can be further decomposed, they basically assume that $K'_{M-1} + K_M = K_{M-1}$. Now, let us compare these two different ways of aggregation and their affect on measuring productivity:

$$K_{M-1} = K'_{M-1} + K_M; \tag{A.28}$$

$$K_{M-1} = \left[\left(\frac{\alpha'_{M-1}}{\alpha_{M-1}}\right)^{\frac{1}{\gamma}} K'^{\frac{\gamma-1}{\gamma}}_{M-1} + \left(\frac{\alpha_M}{\alpha_{M-1}}\right)^{\frac{1}{\gamma}} K_M^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{A.29}$$

where $\left(\frac{\alpha'_{M-1}}{\alpha_{M-1}}\right) + \left(\frac{\alpha_M}{\alpha_{M-1}}\right) = 1$. Following our proposition 1, we can see that until $\gamma \to \infty$, the CES aggregation is always smaller than linear summation. Hence, we conclude that $A \leq A'$, which eventually leads to our proposition 3.

A.7 Proof of Proposition 2

(1) AE in a Multi-sector Economy with Sector-specific Factors Formally we need to show the following statement:
For a multi-sector economy with S distinct sectors and only sector-specific factors , consider a dataset $\mathcal{D}_s = (R_s, L_s, \{K_{ms}\}_{m=1}^{M_s})_{N_s}$ $\forall s \in S$, and fixed model parameters for each sector $\mathcal{P}_s = (\sigma_s, \alpha_s, \{\alpha_{ms}\}_{m=1}^{M_s})$: Under Convention 1, the the total allocative efficiency in the multi-sector economy AE calculated from a model with homogeneous capital ($\gamma_s \to \infty$) is larger than the AE calculated from a model with heterogeneous capital with sector-specific elasticity of substitution (any finite γ_s):

$$AE(\{\gamma_s\}_{s=1}^{S}, \{\mathcal{D}_s\}_{s=1}^{S}, \{\mathcal{P}_s\}_{s=1}^{S}) < \lim_{\gamma_s \to \infty, \forall s} AE(\{\gamma_s\}_{s=1}^{S}, \{\mathcal{D}_s\}_{s=1}^{S}, \{\mathcal{P}_s\}_{s=1}^{S})$$

We can easily establish this by applying Proposition 1 to this economy. Under Convention 1 and sector-specific factor supply, which implies no aggregate wedges, the Allocative Efficiency AE_s for each sector s calculated from a model with homogeneous capital ($\gamma_s \to \infty$) is larger than the AE_s calculated from a model with heterogeneous capital with sector-specific elasticity of substitution (any finite γ_s):

$$AE_s(\gamma_s, \mathcal{D}_s, \mathcal{P}_s) < \lim_{\gamma_s \to \infty} AE_s(\gamma_s, \mathcal{D}_s, \mathcal{P}_s)$$

for all $s = 1, \ldots, S$, as long as γ_s is well-defined for each sector s. Further, the overall allocative efficiency in the multi-sector economy AE is given by

$$AE = \frac{Y}{Y^e} = \Pi_{s=1}^{S} AE_s^{\theta_s}$$

. Using the sectoral AE inequalities implies that the product of the larger sides of the ineuqlaitieis must be larger than the product of the samller sides of the inequalities:

$$\prod_{s=1}^{S} AE_s(\gamma_s, \mathcal{D}_s, \mathcal{P}_s) < \lim_{\gamma_s \to \infty, \forall s} \prod_{s=1}^{S} AE_s(\gamma_s, \mathcal{D}_s, \mathcal{P}_s)$$

This means we must have:

$$AE(\{\gamma_s\}_{s=1}^{S}, \{\mathcal{D}_s\}_{s=1}^{S}, \{\mathcal{P}_s\}_{s=1}^{S}) < \lim_{\gamma_s \to \infty, \forall s} AE(\{\gamma_s\}_{s=1}^{S}, \{\mathcal{D}_s\}_{s=1}^{S}, \{\mathcal{P}_s\}_{s=1}^{S})$$

(2) AE in Multi-sector Economy with Production Network Except for the multi-sector setting, we now adding production network in our framework, in the spirit of Hang, Krishna and Tang (2020). For each sector s, market clearing for sectoral output implies

$$Y_s = C_s + M_s \tag{A.30}$$

where M_s is the intermediate input that sector s provides for productions of rest sectors.

The final product Y is produced by a representative firms according to

$$Y = \prod_{s=1}^{S} C_s^{\theta_s}, \text{ with } \sum_{s=1}^{S} \theta_s = 1 \tag{A.31}$$

Lastly, firm i's production function now is given by:

$$Y_{si} = A_{si} \left[\left(\sum_{m=1}^{M} \alpha_{sm}^{\frac{1}{\gamma_s}} K_{smi}^{\frac{\gamma_s - 1}{\gamma_s}} \right)^{\frac{\gamma_s}{\gamma_s - 1}\alpha_s} L_{si}^{1-\alpha_s} \right]^{1-\sigma_s} \cdot \left(\prod_{q=1}^{S} M_{qsi}^{\lambda_{qs}} \right)^{1-\delta_s} , \forall M \in \{1, 2, 3, \ldots\} \tag{A.32}$$

where M_{qsi} denotes the intermediate good that firm i in sector s purchase from sector q. λ_{qs} implies the production share. Following the same logic, we can see that our lemma 2 still holds, since the production network has no correlation with the elasticity of substitution across different types of capital. Hence, our proposition 1, 2 and 3 still hold following lemma 2 with production function. This completes our proof.

A.8 Derivation of the Welfare Cost Formula

From D. R. Baqaee and Farhi (2020), the second-order welfare loss of distortions around the optimal allocation for each sector (sector index s is omitted) is:

$$\begin{aligned} \Delta \log TFP = & \frac{1}{2}\sigma Var_\lambda \left[(1-\alpha)\log(1+\tau_{L_i}) + \alpha\alpha_E \log(1+\tau_{E_i}) + \alpha\alpha_S \log(1+\tau_{S_i}) \right] \\ & + \frac{1}{2}\alpha(1-\alpha) Var_\lambda \left[\alpha_E \log(1+\tau_{E_i}) + \alpha_S \log(1+\tau_{S_i}) - \log(1+\tau_{L_i}) \right] \\ & - \frac{1}{2}\gamma\alpha\alpha_E\alpha_S Var_\lambda \left[\log(1+\tau_{E_i}) - \log(1+\tau_{S_i}) \right] \end{aligned} \tag{A.33}$$

where $\alpha_E = \frac{\alpha_E^{\frac{1}{\gamma}} E^{\frac{\gamma-1}{\gamma}}}{\alpha_E^{\frac{1}{\gamma}} E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S^{\frac{\gamma-1}{\gamma}}}$ and $\alpha_S = \frac{\alpha_S^{\frac{1}{\gamma}} S^{\frac{\gamma-1}{\gamma}}}{\alpha_E^{\frac{1}{\gamma}} E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S^{\frac{\gamma-1}{\gamma}}}$ are the equilibrium expenditure share on E and S in efficient equilibrium, and $\lambda_i = \frac{P_i Y_i}{PY}$ is the sales share/ Domar weight of firm i.

Rearranging terms yields

$$\begin{aligned} \Delta \log TFP = & \frac{(\sigma-1)\alpha^2\alpha_E^2 + \alpha(\alpha_E^2 + \gamma\alpha_E\alpha_S)}{2} Var_\lambda \left[\log(1+\tau_{E_i}) \right] \\ & + \frac{(\sigma-1)\alpha^2\alpha_S^2 + \alpha(\alpha_S^2 + \gamma\alpha_E\alpha_S)}{2} Var_\lambda \left[\log(1+\tau_{S_i}) \right] \\ & - \alpha\alpha_E\alpha_S \left(\gamma - \sigma\alpha - (1-\alpha) \right) Cov_\lambda \left[\log(1+\tau_{E_i}), \log(1+\tau_{S_i}) \right], \end{aligned}$$

and aggregating across sectors yields the formula in 3.

A.9 Welfare Costs in the Structural Model

In the log-linearized model, in the efficient allocation in the model's stochastic steady state, we have $a \sim N(0, \sigma_a^2)$. Therefore, we have:

$$\lambda_i = \frac{P_i Y_i}{\sum_i P_i Y_i} = \frac{A_i^{\sigma-1}}{\mathbb{E}_i A_i^{\sigma-1}} = \frac{\exp[(\sigma-1)a_i]}{\mathbb{E}_i[\exp[(\sigma-1)a_i]]} = \frac{\exp[(\sigma-1)a_i]}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}$$

With both $mrpe_i$ and $mrps_i$ are log-normally distributed and correlated with a_i, we now compute:

$$\begin{aligned}
\mathbb{E}_\lambda[mrpe_i^2] &= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\mathbb{E}[\exp[(\sigma-1)a_i]mrpe_i^2] \\
&= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\int \frac{mrpe^2 e^{(\sigma-1)a}}{2\pi\sqrt{\sigma_a^2\sigma_{mrpe}^2 - 2\sigma_{mrpe,a}}} e^{-\frac{\left(\sigma_{mrpe}^2 a^2 + \sigma_a^2 mrpe^2 - 2\sigma_{mrpe,a} a \cdot mrpe\right)}{2(\sigma_a^2\sigma_{mrpe}^2 - 2\sigma_{mrpe,a})}} (da)(dmrpe) \\
&= \sigma_{mrpe}^2 + (\sigma-1)^2\sigma_{mrpe,a}^2
\end{aligned}$$

Similarly, we have that:

$$\begin{aligned}
\mathbb{E}_\lambda[mrpe_i] &= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\mathbb{E}[\exp[(\sigma-1)a_i]mrpe_i] \\
&= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\int \frac{mrpe \cdot e^{(\sigma-1)a}}{2\pi\sqrt{\sigma_a^2\sigma_{mrpe}^2 - 2\sigma_{mrpe,a}}} e^{-\frac{\left(\sigma_{mrpe}^2 a^2 + \sigma_a^2 mrpe^2 - 2\sigma_{mrpe,a} a \cdot mrpe\right)}{2(\sigma_a^2\sigma_{mrpe}^2 - 2\sigma_{mrpe,a})}} (da)(dmrpe)) \\
&= (\sigma-1)\sigma_{mrpe,a}
\end{aligned}$$

Therefore, $Var_\lambda[mrpe]$can be written as:

$$Var_\lambda[mrpe] = \mathbb{E}_\lambda[mrpe^2] - (\mathbb{E}_\lambda[mrpe])^2 = \sigma_{mrpe}^2$$

Similarly, we can write

$$Var_\lambda[mrps] = \mathbb{E}_\lambda[mrps^2] - (\mathbb{E}_\lambda[mrps])^2 = \sigma_{mrps}^2$$

Similarly, we compute the covariance:

$$Cov_\lambda[mrpe, mrps] = \mathbb{E}_\lambda[mrpe \cdot mrps] - \mathbb{E}_\lambda[mrpe]\mathbb{E}_\lambda[mrps]$$

where

$$
\begin{aligned}
\mathbb{E}_\lambda[mrpe_i mrps_i] &= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\mathbb{E}[\exp[(\sigma-1)a_i]mrpe_i mrps_i] \\
&= \frac{1}{\exp[\frac{1}{2}(\sigma-1)^2\sigma_a^2]}\int mrpe \cdot mrps \cdot e^{(\sigma-1)a} f(a, mrpe, mrps)(da)(dmrpe)(dmrps) \\
&= \sigma_{mrpe,mrps} + (\sigma-1)^2\sigma_{mrpe,a}\sigma_{mrps,a}
\end{aligned}
$$

and

$$\mathbb{E}_\lambda[mrpe]\mathbb{E}_\lambda[mrps] = (\sigma-1)^2\sigma_{mrpe,a}\sigma_{mrps,a}.$$

Simplifying yields

$$Cov_\lambda[mrpe, mrps] = \sigma_{mrpe,mrps}$$

Therefore, in the structural model, we have that:

$$
\begin{aligned}
\Delta \log TFP = &\frac{(\sigma-1)\alpha^2\alpha_E^2 + \alpha(\alpha_E^2 + \gamma\alpha_E\alpha_S)}{2}\sigma_{mrpe}^2 \\
&+ \frac{(\sigma-1)\alpha^2\alpha_S^2 + \alpha(\alpha_S^2 + \gamma\alpha_E\alpha_S)}{2}\sigma_{mrps}^2 \\
&- \alpha\alpha_E\alpha_S\left(\gamma - \sigma\alpha - (1-\alpha)\right)\sigma_{mrpe,mrps}
\end{aligned}
$$

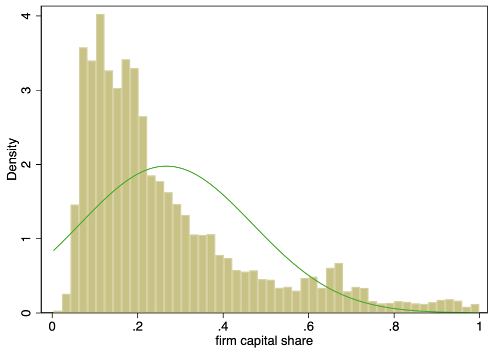
B Details of Data Clean

B.1 Compustat North America

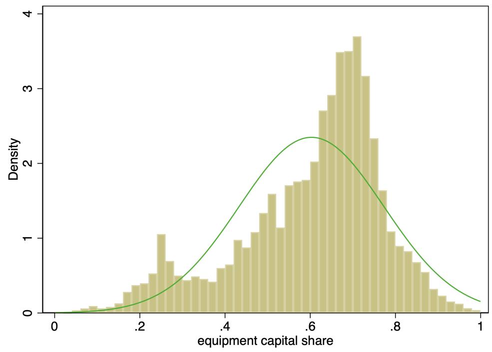
To construct our sample of Compustat North America, we first keep firms in the US (fic == USA) and firms using US dollars (curcd == USD). We then keep observations that have a 3-digit NAICS number for the sector identifier. Next, we drop observations with repeated firm-year pairs. To calibrate the capital share, we link 3-digit NAICS to BEA sector codes and then use BEA sector shares as the sector-specific firm capital share. To calibrate the equipment and structures share, we define the equipment share as the ratio of equipment (FATE) to total fixed assets (PPEGT), and the structures share as one minus the equipment share. Structures are calculated as total fixed assets minus equipment.

Moreover, we trim our sample using ARPK, ARPE, ARPS, ARPL, capital-labor ratio, and structures-equipment ratio. We trim 1% from each side of all the above variables in each year. After trimming our sample, we again drop the observations whose equipment or structures shares are above 1 or below -1. We show the distribution of shares in our cleaned sample below.

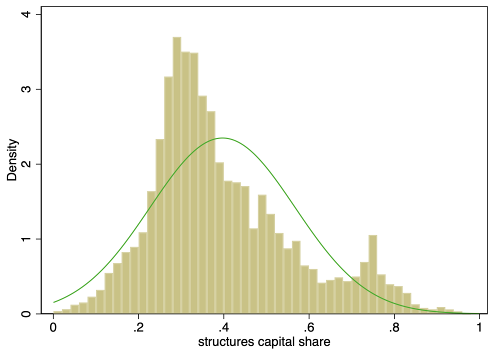
Figure B.1: US Compustat Shares Distribution



Capital



Equipment



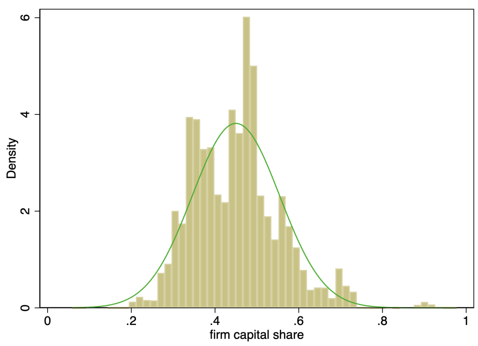
Structures

B.2 Indian ASI

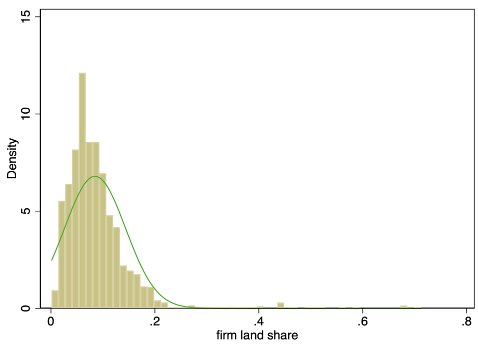
We download the ASI data for each year and append all datasets together to construct a panel dataset. The ASI panel data contains firm-level production data and panel identifiers. We then harmonize all variable and sector definitions across different survey waves. For each type of capital (and their sum), we measure the (nominal) net stock of capital by using the average of the net opening and net closing values. Note that we generate both (average) gross and (average) net capital values. However, we use net capital values in our analysis due to better data coverage. Also note that before 2001, there is no ”Category 6: pollution control assets” collected in the dataset. The harmonized capital variables include the average net value of land, buildings, plant and machinery, computer equipment, pollution equipment, and all capital assets. We also generate total sales, total material costs, total labor, and total wages. [44]

To clean the sample, we first generate “capital others” as the total fixed assets minus all other five types of capital. We merge the data with MOSPI to obtain the capital share. Then, to trim the data, we compute the average revenue product of all six types of capital and trim the data at 1% on both sides in each year. The distributions of each type of capital are also shown below.

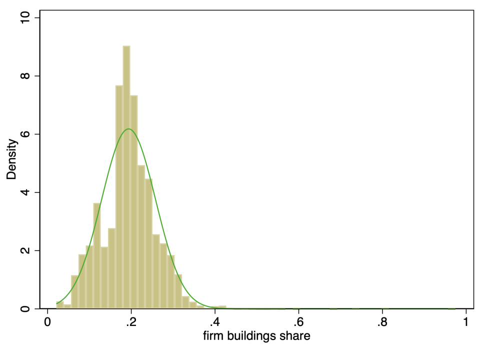
Figure B.2: Indian ASI Shares Distribution



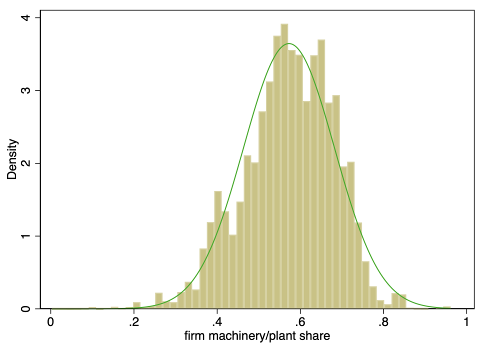
Capital



Land

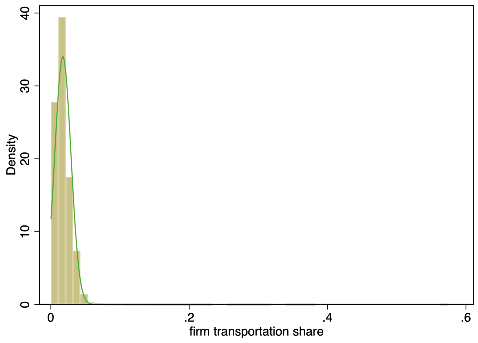


Buildings

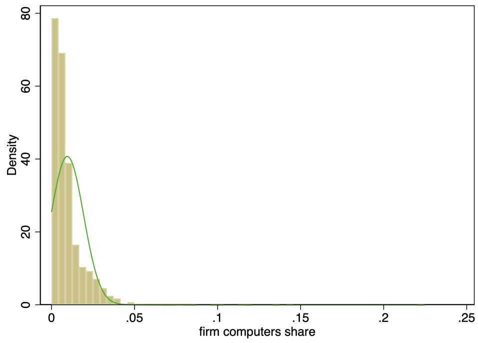


Machinery and Plants

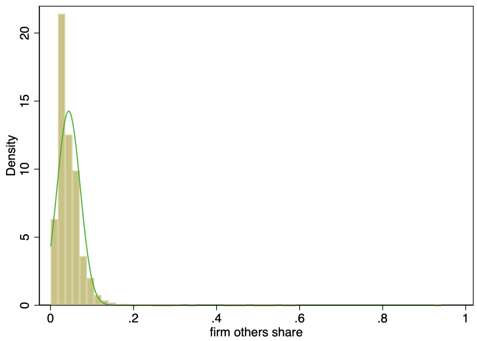
[44] To harmonize sectoral concordance, we use NIC98 (for data before 2004), NIC04 (for data from 2004 to 2007), and NIC08 (for data after 2007).



Transportation



Computers



Others

C Elasticity of Equipment and Structure Substitution Estimation

We provide some alternative specifications of the estimation of the elasticity of equipment and structure substitution. First, we try to use the book values of equipment and structures instead of the quantity value from perpetual inventory method. Specifically, we run the following regression

$$\ln\left(\frac{\hat{R}^S_{i(f)t} P^S_{i(f)t-1} S_{ft}}{\hat{R}^E_{i(f)t} \underbrace{P^E_{i(f)t-1} E_{ft}}_{\text{Book Value}}}\right) = (\gamma - 1)\ln\left(\frac{R_{Edt}}{R_{Sdt}}\right) + FEs \tag{C.1}$$

The usage of book value variables are closer to our empirical framework. We also test if different fixed effects specifications affect our estimation. Moreover, we also test if using both equipment and structures shift share as IV, or only equipment or structures as the source of exogenous variation. Finally, we also test if the long-run estimation will change our results. All alternative results are attached below and our results are qualitatively stable.

We also test if different fixed effects will deliver various results

Table C.1: Robustness of Stock and Expenditure Measures

	(1) $\ln \frac{BV_{Sit}}{BV_{Eit}}$	(2) IV	(3) $\ln \frac{\hat{R}_{Sdt}P_{Sdt}K_{Sdit}}{\hat{R}_{Edt}P_{Edt}K_{Edit}}$	(4) IV
log_re_rs	-0.10876***	-0.36460***	-0.70582***	-0.31172***
	(0.02163)	(0.06799)	(0.02232)	(0.06933)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	82,104	81,496	82,064	81,460
R^2	0.788	-0.006	0.818	0.036

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.2: Robustness of Fixed Effects

	(1)	(2)	(3)
log_re_rs	0.31282***	0.56414***	0.57555***
	(0.06955)	(0.11109)	(0.11705)
Firm FE	Yes	Yes	Yes
Year FE	Yes	No	No
NAICS-1 digit by Year FE	No	Yes	Yes
State by Year FE	No	No	Yes
Observations	81,287	81,287	81,179
R^2	0.001	-0.006	-0.006

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.3: Robustness of the Shifts in IV

	(1) E-S shifts	(2)Only E price shifts	(3) Only S price shifts
log_re_rs	0.31282***	0.37938***	0.20444**
	(0.06955)	(0.07499)	(0.10361)
Firm FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Observations	81,287	81,884	81,884
R^2	0.001	-0.002	0.004

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.4: Long-run Elasticity of Capital Substitution: using stock measure

	(1)	(2)	(3)	(4)	(5)
log_re_rs	0.31282***	0.13005***	0.26455***	0.32323***	0.20520**
	(0.06955)	(0.02863)	(0.05535)	(0.07238)	(0.09292)
L.log_quantity_stock_s_e		0.62520***			
		(0.00792)			
L3.log_quantity_stock_s_e			0.28806***		
			(0.01131)		
L5.log_quantity_stock_s_e				0.12281***	0.13332***
				(0.01282)	(0.01894)
L10.log_quantity_stock_s_e					-0.04257***
					(0.01567)
Firm FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Observations	81,287	69,746	53,276	41,978	22,706
R^2	0.001	0.439	0.105	0.019	0.020

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

D Solving the Baseline Structure Model

D.1 Solving the Firm's Problem

In this section, we provide detail process of solving out structure model. we assume that now equipment and structures are combined as a capital bundle in a CES fashion:

$$Y_{it} = \hat{A}_{it} \left(\alpha_E^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\hat{\alpha}_K} N_{it}^{\hat{\alpha}_N} \tag{D.1}$$

The firm's labor decision is unchanged compared to the Cobb-Douglas case. Hence, after optimizing labor choice, firm's problem is given by

$$\mathcal{V}\left(E_{it}, S_{it}, \mathcal{I}_{it}\right) = \max_{E_{i,t+1}, S_{i,t+1}} \mathbb{E}_{it} \Bigg[G A_{it} \left(\alpha_E^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\alpha} - T_{i,t+1}^{E} E_{i,t+1}(1-\beta(1-\delta_E))$$
$$- \Phi\left(E_{i,t+1}, E_{it}\right) - T_{i,t+1}^{S} S_{i,t+1}(1-\beta(1-\delta_S)) - \Phi\left(S_{i,t+1}, S_{it}\right) \Bigg] + \beta \mathbb{E}_{it}\left[\mathcal{V}\left(E_{i,t+1}, S_{i,t+1}, \mathcal{I}_{it+1}\right)\right]$$

where $\alpha \equiv \frac{\alpha_K}{1-\alpha_N}$, $\alpha_K = \left(1 - \frac{1}{\theta}\right) \hat{\alpha}_N$, $\alpha_N = \left(1 - \frac{1}{\theta}\right) \hat{\alpha}_N$ and $A_{it} \equiv \hat{A}_{it}^{\frac{1-\frac{1}{\theta}}{1-\alpha_N}}$. The term $G \equiv (1-\alpha_N) \left(\frac{\alpha_N}{W}\right)^{\frac{\alpha_N}{1-\alpha_N}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_N}}$ is a constant when there is no aggregate risk. Combine Euler

Equation and Envo Thm:

$$[E_{it}] : \mathbb{E}_{it}\left[\beta\Pi_1\left(E_{it+1}, S_{it+1}A_{it+1}\right) - \beta\Phi_2\left(E_{it+2,}E_{it+1}\right) - T^E_{it+1}\cdot\left[1-\beta\left(1-\delta_E\right)\right] - \Phi_1\left(E_{it+1}, E_{it}\right)\right] = 0 \tag{D.2}$$

$$[s_{it}] : \mathbb{E}_{it}\left[\beta\Pi_2\left(E_{it+1}, S_{it+1}, A_{it+1}\right) - \beta\Phi_2\left(S_{it+2}, S_{it+1}\right) - T^S_{it+1}\cdot\left[1-\beta\left(1-\delta_S\right)\right] - \Phi_1\left(S_{it+1}, S_{it}\right)\right] = 0 \tag{D.3}$$

Steady state: we have

- $A_{it} = 1$ and $T^S = T^E = 1$
- $\overline{\Pi}_1(E,S,A) = GA\alpha\alpha_E^{\frac{1}{\gamma}}E^{-\frac{1}{\gamma}}\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1}$
- $\overline{\Pi}_2(E,S,A) = GA\alpha\alpha_S^{\frac{1}{\gamma}}S^{-\frac{1}{\gamma}}\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1}$
- $\overline{\Phi}_1(E) = \hat{\xi}_E\delta_E$, $\overline{\Phi}_1(S) = \hat{\xi}_S\delta_S$
- $\overline{\Phi}_2\left(E\right) = \frac{\hat{\xi}_E}{2}\left(1-\delta_E\right)^2 - \frac{\hat{\xi}_E}{2}$, $\overline{\Phi}_2\left(S\right) = \frac{\hat{\xi}_S}{2}\left(1-\delta_S\right)^2 - \frac{\hat{\xi}_S}{2}$

Plug them back:

$$\beta\cdot G\alpha\alpha_E^{\frac{1}{\gamma}}E^{-\frac{1}{\gamma}}\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1} - \beta\left(\frac{\hat{\xi}_E}{2}\left(1-\delta_E\right)^2 - \frac{\hat{\xi}_E}{2}\right) - \left[1-\beta(1-\delta_E)\right] - \hat{\xi}_E\delta_E = 0$$

$$\beta\cdot G\alpha\alpha_S^{\frac{1}{\gamma}}S^{-\frac{1}{\gamma}}\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1} - \beta\left(\frac{\hat{\xi}_S}{2}\left(1-\delta_S\right)^2 - \frac{\hat{\xi}_S}{2}\right) - \left[1-\beta(1-\delta_S)\right] - \hat{\xi}_S\delta_S = 0$$

In the steady state,

$$Y = Y_i = \left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\hat{\alpha}_K} N^{\hat{\alpha}_N}\Big|_{N=1,P=1} = \left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\hat{\alpha}_K} \tag{D.4}$$

So the wage will be given by

$$W = \hat{\alpha}_N\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\hat{\alpha}_K} \tag{D.5}$$

Hence,

$$G = (1-\alpha_N)\left(\frac{\alpha_N}{W}\right)^{\frac{\alpha_N}{1-\alpha_N}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_N}} = (1-\alpha_N)\left(\alpha_E^{\frac{1}{\gamma}}E^{\frac{\gamma-1}{\gamma}} + \alpha_S^{\frac{1}{\gamma}}S^{\frac{\gamma-1}{\gamma}}\right)^{\alpha_K\frac{\gamma}{\gamma-1}\frac{\frac{1}{\theta}-\alpha_N}{1-\alpha_N}}$$

We can easily see that the Euler equations are the same as the Cobb-Douglas function case

$$[E_{it}]: \beta\overline{\Pi}_1\widetilde{\pi}_1\left(E_{it+1},S_{it+1},A_{it+1}\right)-\beta\overline{\Phi}_2(E)\widetilde{\phi}_2\left(E_{it+2},E_{it+1}\right)-\left[1-\beta\left(1-\delta_E\right)\right]\widetilde{\tau}^E_{i,t+1}-\overline{\Phi}_1(E)\widetilde{\phi}_1\left(E_{it+1},E_{it}\right)=0 \tag{D.6}$$

$$[S_{it}]: \beta\overline{\Pi}_2\widetilde{\pi}_2\left(E_{it+1},S_{it+1},A_{it+1}\right)-\beta\overline{\Phi}_2(S)\widetilde{\phi}_2\left(S_{it+2},S_{it+1}\right)-[1-\beta(1-\delta_S)]\widetilde{\tau}^S_{i,t+1}-\overline{\Phi}_1(S)\widetilde{\phi}_1\left(S_{it+1},S_{it}\right)=0 \tag{D.7}$$

except for constants $\overline{\Pi}_1$ and $\overline{\Pi}_2$ and variables $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$. Under the CES production function, the two steady state variables are given by

$$\overline{\Pi}_1 = GA\alpha\alpha_E E^{-\frac{1}{\gamma}}\left(\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1} \tag{D.8}$$

$$\overline{\Pi}_2 = GA\alpha\alpha_S S^{-\frac{1}{\gamma}}\left(\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}\alpha-1} \tag{D.9}$$

Now we apply log linearization on Π_1 and Π_2 to get $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$. First, for $\widetilde{\pi}_1$

$$\begin{aligned}
\log\left(\Pi_1\left(E_{i,t+1},S_{i,t+1},A_{i,t+1}\right)\right) &= \log\left(G\alpha\alpha_E\right)+\log\left(A_{i,t+1}\right)-\frac{1}{\gamma}\log\left(E_{it+1}\right)\\
&+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\log\left(\alpha_E E_{it+1}^{\frac{\gamma-1}{\gamma}}+\alpha_S S_{it+1}^{\frac{\gamma-1}{\gamma}}\right)\\
\text{LHS:} &= \log\left(\bar{\Pi}_1\right)+\widetilde{\pi}_1\left(E_{it+1},S_{it+1},A_{it+1}\right)\\
\text{RHS:} &= \log\left(G\alpha\alpha_E A\right)+\widetilde{a}_{i,t+1}-\frac{1}{\gamma}\left[\log(E)+\widetilde{e}_{it+1}\right]\\
&+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\left[\log\left(\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}\right)+\frac{\alpha_E\frac{\gamma-1}{\gamma}E^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}\widetilde{e}_{it+1}\right.\\
&\left.+\frac{\alpha_S\frac{\gamma-1}{\gamma}S^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}\widetilde{s}_{it+1}\right]
\end{aligned}$$

Hence,

$$\begin{aligned}
\beta\overline{\Pi}_1\widetilde{\pi}_1\left(E_{it+1},S_{it+1},A_{it+1}\right) =&\beta\overline{\Pi}_1\left[\widetilde{a}_{i,t+1}+\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\frac{\alpha_E\frac{\gamma-1}{\gamma}E^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}-\frac{1}{\gamma}\right)\widetilde{e}_{it+1}\right.\\
&\left.+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\frac{\alpha_S\frac{\gamma-1}{\gamma}S^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}\widetilde{s}_{it+1}\right]
\end{aligned} \tag{D.10}$$

And similarly, we can derive $\widetilde{\pi}_2$ as

$$\begin{aligned}
\beta\overline{\Pi}_2\widetilde{\pi}_2\left(E_{it+1},S_{it+1},A_{it+1}\right) =&\beta\overline{\Pi}_2\left[\widetilde{a}_{i,t+1}+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\frac{\alpha_E\frac{\gamma-1}{\gamma}E^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}\widetilde{e}_{it+1}\right.\\
&\left.+\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\frac{\alpha_S\frac{\gamma-1}{\gamma}S^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}}+\alpha_S S^{\frac{\gamma-1}{\gamma}}}-\frac{1}{\gamma}\right)\widetilde{s}_{it+1}\right]
\end{aligned} \tag{D.11}$$

For simplicity, we just denote that $\Gamma_S = \frac{\alpha_S \frac{\gamma-1}{\gamma} S^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}} + \alpha_S S^{\frac{\gamma-1}{\gamma}}}$ and $\Gamma_E = \frac{\alpha_E \frac{\gamma-1}{\gamma} E^{\frac{\gamma-1}{\gamma}}}{\alpha_E E^{\frac{\gamma-1}{\gamma}} + \alpha_S S^{\frac{\gamma-1}{\gamma}}}$. With all we've gotten above, now the firm's Euler equations are given by

$$\begin{aligned} [E_{it}] : \mathbb{E}_{it} \Bigg\{ & \beta \overline{\Pi}_1 \left[\widetilde{a}_{i,t+1} + \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_E - \frac{1}{\gamma} \right) \widetilde{e}_{it+1} + \left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_S \widetilde{s}_{it+1} \right] \\ & + \beta \hat{\xi}_E \left(\widetilde{e}_{it+2} - \widetilde{e}_{it+1} \right) - \left[1 - \beta \left(1 - \delta_E \right) \right] \widetilde{\tau}^E_{i,t+1} - \hat{\xi}_E \left(\widetilde{e}_{i,t+1} - \widetilde{e}_{it} \right) \Bigg\} = 0 \end{aligned} \tag{D.12}$$

$$\begin{aligned} [S_{it}] : \mathbb{E}_{it} \Bigg\{ & \beta \overline{\Pi}_2 \left[\widetilde{a}_{i,t+1} + \left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_E \widetilde{e}_{it+1} + \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_S - \frac{1}{\gamma} \right) \widetilde{s}_{it+1} \right] \\ & + \beta \hat{\xi}_S \left(\widetilde{s}_{it+2} - \widetilde{s}_{it+1} \right) - \left[1 - \beta \left(1 - \delta_S \right) \right] \widetilde{\tau}^S_{i,t+1} - \hat{\xi}_S \left(\widetilde{s}_{i,t+1} - \widetilde{s}_{it} \right) \Bigg\} = 0 \end{aligned} \tag{D.13}$$

After normalized by $\beta\overline{\Pi}_1$ and $\beta\overline{\Pi}_2$, we have

$$\begin{aligned} \widetilde{e}_{i,t+1} \left[(1+\beta)\xi_E - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_E - \frac{1}{\gamma} \right) \right] &= \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \mathbb{E}_{it}(\widetilde{\tau}^e_{it+1}) + \beta\xi_E \mathbb{E}_{it}(\widetilde{e}_{i,t+2}) \\ &+ \left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_S \widetilde{s}_{i,t+1} + \xi_E \widetilde{e}_{i,t} \end{aligned} \tag{D.14}$$

$$\begin{aligned} \widetilde{s}_{i,t+1} \left[(1+\beta)\xi_S - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_S - \frac{1}{\gamma} \right) \right] &= \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \mathbb{E}_{it}(\widetilde{\tau}^s_{it+1}) + \beta\xi_S \mathbb{E}_{it}(\widetilde{s}_{i,t+2}) \\ &+ \left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_E \widetilde{e}_{i,t+1} + \xi_S \widetilde{s}_{i,t} \end{aligned} \tag{D.15}$$

We can verify that, when $\gamma \to 1$

$$\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_E - \frac{1}{\gamma} \to \alpha\alpha_E - 1$$
$$\left(\frac{\gamma}{\gamma-1}\alpha - 1 \right) \Gamma_S - \frac{1}{\gamma} \to \alpha\alpha_S - 1$$

and this whole system converges back to the Cobb-Douglas case. Eventually, when we very the elasticity of capital substitution, the curvature of structures and equipment change. This allows us to use the same methodology to solve the whole model. The system of solving all parameters in the equipment policy functions is given by

$$[1,\widetilde{e}_{it}]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_1^E=\beta\xi_E\left[\left(\psi_1^E\right)^2+\psi_2^E\psi_2^S\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\cdot\psi_2^S+\xi_E$$
$$[2,\widetilde{s}_{it}]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_2^E=\beta\xi_E\left[\psi_1^E\cdot\psi_2^E+\psi_1^S\cdot\psi_2^E\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\cdot\psi_1^S$$
$$[3,\mathbb{E}_{it}(a_{i,t+1})]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right](1+\gamma_E)\cdot\psi_3^E=(1+\gamma_E)+\beta\xi_E\Big[\psi_1^E\psi_3^E\left(1+\gamma_E\right)$$
$$+\psi_2^E\psi_3^S\left(1+\gamma_S\right)+\psi_3^E(1+\gamma_E)\rho\Big]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S(1+\gamma_S)\cdot\psi_3^S$$
$$[4,\varepsilon_{i,t+1}^E]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_4^E=1+\beta\xi_E\left(\psi_1^E\psi_4^E+\psi_2^E\psi_5^S\right)+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\psi_5^S$$
$$[5,\varepsilon_{i,t+1}^S]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_5^E=\beta\xi_E\left(\psi_1^E\psi_5^E+\psi_2^E\psi_4^S\right)+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\psi_4^S$$
$$[6,\chi_i^E]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_6^E=1+\beta\xi_E\left[\psi_1^E\psi_6^E+\psi_2^E\psi_7^S+\psi_6^E\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\psi_7^S$$
$$[7,\chi_i^S]:\left[(1+\beta)\xi_E-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E-\frac{1}{\gamma}\right)\right]\cdot\psi_7^E=\beta\xi_E\left[\psi_1^E\psi_7^E+\psi_2^E\psi_6^S+\psi_7^E\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S\psi_6^S$$

Similarly, for S:

$$[1,\widetilde{s}_{it}]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_1^S=\beta\xi_S\left[\left(\psi_1^S\right)^2+\psi_2^E\psi_2^S\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_2^E+\xi_S$$
$$[2,\widetilde{e}_{it}]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_2^S=\beta\xi_S\left[\psi_1^S\psi_2^S+\psi_1^E\psi_2^S\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_1^E$$
$$[3,\mathbb{E}_{it}\left(a_{i,t+1}\right)]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right](1+\gamma_S)\cdot\psi_3^S=(1+\gamma_S)+\beta\xi_S\Big[\rho\psi_3^S(1+\gamma_S)$$
$$+\psi_2^S\psi_3^E(1+\gamma_E)+\psi_1^S\psi_3^S(1+\gamma_S)\Big]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E(1+\gamma_E)\cdot\psi_3^E$$
$$[4,\varepsilon_{i,t+1}^S]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_4^S=1+\beta\xi_S\left(\psi_1^S\psi_4^S+\psi_2^S\psi_5^E\right)+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_5^E$$
$$[5,\varepsilon_{i,t+1}^E]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_5^S=\beta\xi_S\left(\psi_1^S\psi_5^S+\psi_2^S\psi_4^E\right)+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_4^E$$
$$[6,\chi_i^S]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_6^S=1+\beta\xi_S\left[\psi_1^S\psi_6^S+\psi_2^S\psi_7^E+\psi_6^S\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_7^E$$
$$[7,\chi_i^E]:\left[(1+\beta)\xi_S-\left(\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_S-\frac{1}{\gamma}\right)\right]\cdot\psi_7^S=\beta\xi_S\left[\psi_1^S\psi_7^S+\psi_2^S\psi_6^E+\psi_7^S\right]+\left(\frac{\gamma}{\gamma-1}\alpha-1\right)\Gamma_E\cdot\psi_6^E$$

We first use $[1,E]$, $[1,S]$, $[2,E]$, $[2,S]$ to solve ψ_1^E, ψ_2^E, ψ_1^S and ψ_2^S. Then, using $[2,E]$ and $[3,S]$ to pin down ψ_3^E, ψ_3^S. Next, we use $[4,E]$, $[5,S]$, $[4,E]$, $[5,S]$ to solve ψ_4^E, ψ_5^E, ψ_4^S and ψ_5^S. Then the rest of four equations unused can pin down ψ_6^E, ψ_7^E, ψ_6^S and ψ_7^S. In this way, we do not need to solve 14 different equations in the same time, which provide more loose situation for identification.

Solving procedure:

(1) We solve ψ_1^E, ψ_2^E, ψ_1^S and ψ_2^S first. There are no closed form solutions.

(2) Given ψ_1^E, ψ_2^E, ψ_1^S and ψ_2^S, we can use E3 and S3 to solve explicitly:

$$\psi_3^E = \frac{D_2 + M_1}{D_1 D_2 - M_1 M_2},\ \psi_3^S = \frac{D_1 + M_2}{D_1 D_2 - M_1 M_2} \tag{D.16}$$

where

$$D_1 = (1+\beta)\xi_E - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_E - \frac{1}{\gamma}\right) - \beta\xi_E\psi_1^E - \beta\xi_E\rho \tag{D.17}$$

$$D_2 = (1+\beta)\xi_S - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_S - \frac{1}{\gamma}\right) - \beta\xi_S\psi_1^S - \beta\xi_S\rho \tag{D.18}$$

$$M_1 = \left(\beta\xi_E\psi_2^E + \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_S\right)\frac{1+\gamma_S}{1+\gamma_E} \tag{D.19}$$

$$M_2 = \left(\beta\xi_S\psi_2^S + \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_E\right)\frac{1+\gamma_E}{1+\gamma_S} \tag{D.20}$$

(3) Given ψ_1^E, ψ_2^E, ψ_1^S and ψ_2^S, we can use E4, S4, E5 and S5 to solve explicitly:

$$\psi_4^E = \frac{H_2}{H_1 H_2 - Q_1 Q_2},\ \psi_5^S = \frac{Q_2}{H_1 H_2 - Q_1 Q_2},\ \psi_4^S = \frac{H_1}{H_1 H_2 - Q_1 Q_2},\ \psi_5^E = \frac{Q_1}{H_1 H_2 - Q_1 Q_2} \tag{D.21}$$

where

$$H_1 = (1+\beta)\xi_E - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_E - \frac{1}{\gamma}\right) - \beta\xi_E\psi_1^E \tag{D.22}$$

$$H_2 = (1+\beta)\xi_S - \left(\left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_S - \frac{1}{\gamma}\right) - \beta\xi_S\psi_1^S \tag{D.23}$$

$$Q_1 = \beta\xi_E\psi_2^E + \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_S \tag{D.24}$$

$$Q_2 = \beta\xi_S\psi_2^S + \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_E \tag{D.25}$$

(4) Given ψ_1^E, ψ_2^E, ψ_1^S and ψ_2^S, we can use E6, S6, E7 and S7 to solve explicitly:

$$\psi_6^E = \frac{J_2}{J_1 J_2 - Q_1 Q_2},\ \psi_7^S = \frac{Q_2}{J_1 J_2 - Q_1 Q_2},\ \psi_6^S = \frac{J_1}{J_1 J_2 - Q_1 Q_2},\ \psi_7^E = \frac{Q_1}{J_1 J_2 - Q_1 Q_2} \tag{D.26}$$

where

$$J_1 = \xi_E(1 - \beta\psi_1^E) - \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_E + \frac{1}{\gamma} \tag{D.27}$$

$$J_2 = \xi_S(1 - \beta\psi_1^S) - \left(\frac{\gamma}{\gamma-1}\alpha - 1\right)\Gamma_S + \frac{1}{\gamma} \tag{D.28}$$

D.2 Policy Function Porperties

We use perturbation to solve the model. Specifically, we log-linearize firm's Euler equations around their steady states given by $A_{it} = \bar{A}$ and $T_{it}^E = T_{it}^E = 1$:

$$\widetilde{e}_{i,t+1}\left[(1+\beta)\xi_E + 1 - \alpha_E\alpha\right] = \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \widetilde{\tau}_{it+1}^E + \beta\xi_E\mathbb{E}_{it}(\widetilde{e}_{i,t+2}) + \alpha_S\alpha\widetilde{s}_{i,t+1} + \xi_E\widetilde{e}_{i,t} \quad \text{(D.29)}$$

$$\widetilde{s}_{i,t+1}\left[(1+\beta)\xi_S + 1 - \alpha_S\alpha\right] = \mathbb{E}_{it}(\widetilde{a}_{it+1}) + \widetilde{\tau}_{it+1}^S + \beta\xi_S\mathbb{E}_{it}(\widetilde{s}_{i,t+2}) + \alpha_E\alpha\widetilde{e}_{i,t+1} + \xi_S\widetilde{s}_{i,t} \quad \text{(D.30)}$$

where ξ_E, ξ_S and $\tau_{i,t+1}^E, \tau_{i,t+1}^S$ and rescaled versions of the adjustment cost parameters, $\hat{\xi}_E$, $\hat{\xi}_S$ and the distortion, $\log T_{i,t+1}^E, \log T_{i,t+1}^S$, respectively. We use guess and verify method to solve the two policy functions given below:

$$\widetilde{e}_{i,t+1} = \psi_1^E\widetilde{e}_{it} + \psi_2^E\widetilde{s}_{it} + \psi_3^E\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) + \psi_4^E\varepsilon_{i,t+1}^E + \psi_5^E\varepsilon_{i,t+1}^S + \psi_6^E\chi_i^E + \psi_7^E\chi_i^S \quad \text{(D.31)}$$

$$\widetilde{s}_{i,t+1} = \psi_1^S\widetilde{s}_{it} + \psi_2^S\widetilde{e}_{it} + \psi_3^S\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) + \psi_4^S\varepsilon_{i,t+1}^S + \psi_5^S\varepsilon_{i,t+1}^E + \psi_6^S\chi_i^S + \psi_7^S\chi_i^E \quad \text{(D.32)}$$

where $\psi_1^E \sim \psi_7^E$ and $\psi_1^S \sim \psi_7^S$ are undetermined coefficients and can be pinned down by Euler equations. The solutions of parameters in these policy functions are either no closed form or tedious in math, so our plan is to discuss the intuition of solving this model, and then show how each channel of friction affects future equipment and structures investments.

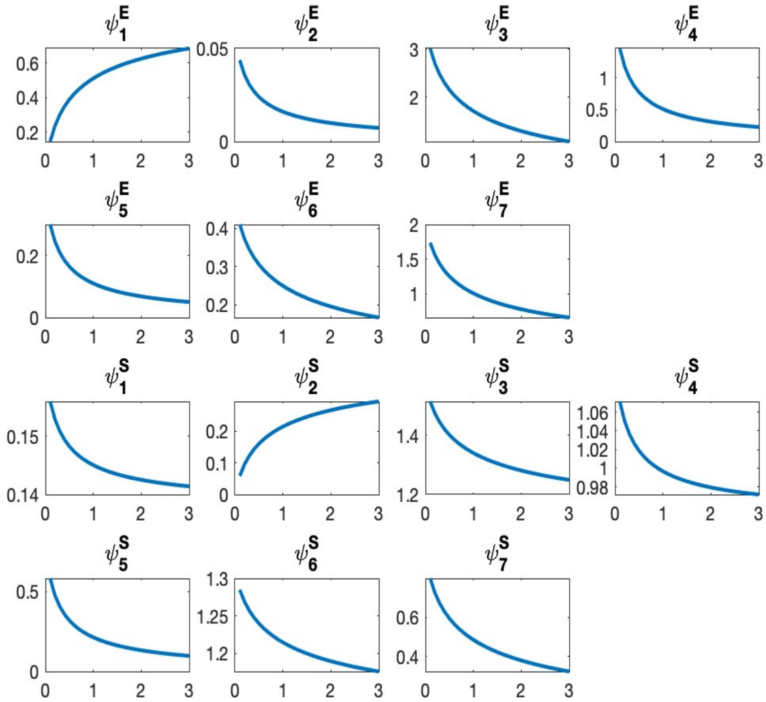
To solve the entire system, the initial step is to determining $\left(\psi_1^E, \psi_2^E, \psi_1^S, \psi_2^S\right)$ simultaneously through a system of quadratic equations. Although there's no closed-form solution for these variables, we understand that they are functions of equipment and structure adjustment costs, (ξ_E, ξ_S). Once these parameters are determined, we can express $\psi_4^E \sim \psi_7^E$ and $\psi_4^S \sim \psi_7^S$ as closed-form solutions derived from them. Finally, ψ_3^E and ψ_3^S can be expressed in closed form with $\left(\psi_1^E, \psi_2^E, \psi_1^S, \psi_2^S\right)$, along with the correlation between tax-like distortions and productivity, (γ_E, γ_S).

How does each individual friction affect future equipment and structure investments? We begin by discussing the two adjustment costs, as they appear to be deterministic in the system, impacting all parameters in the policy functions. We use Figure 10 to illustrate how variations of equipment adjustment costs can affect parameters in the policy functions. Structure adjustment costs operate in a similar manner, and we include them in the Appendix A1[45].

We can see from Figure 10 that, as the cost of adjusting equipment investment (ξ_E) rises gradually from 0.1 to 3, the autocorrelation in equipment investment increases. Firms instead tend to invest more in structure since it is relatively less costly to adjust, leading to a declining correlation between structural stocks in the current and future periods. Meanwhile, ψ_2^E decreases as the equipment stock in $t+1$ is more constrained by the current equipment stock. For the same reason, ψ_2^S increases. Furthermore, we can also see that from Figure 10, $\psi_3^E \sim \psi_7^E$ and $\psi_3^S \sim \psi_7^S$ all decline with increasing equipment adjustment costs. This decline can be attributed to the fact that higher equipment adjustment costs lead to more lumpy firm investments in equipment, resulting in less responsiveness to

[45] For robustness, we randomly generate all other frictions except for ξ_1 or ξ_2, as other frictions theoretically would not affect ψ_1^E, ψ_1^S, ψ_2^E and ψ_2^S.

Figure D.3: How Do Equipment Adjustment Cost Affect Policy Functions?



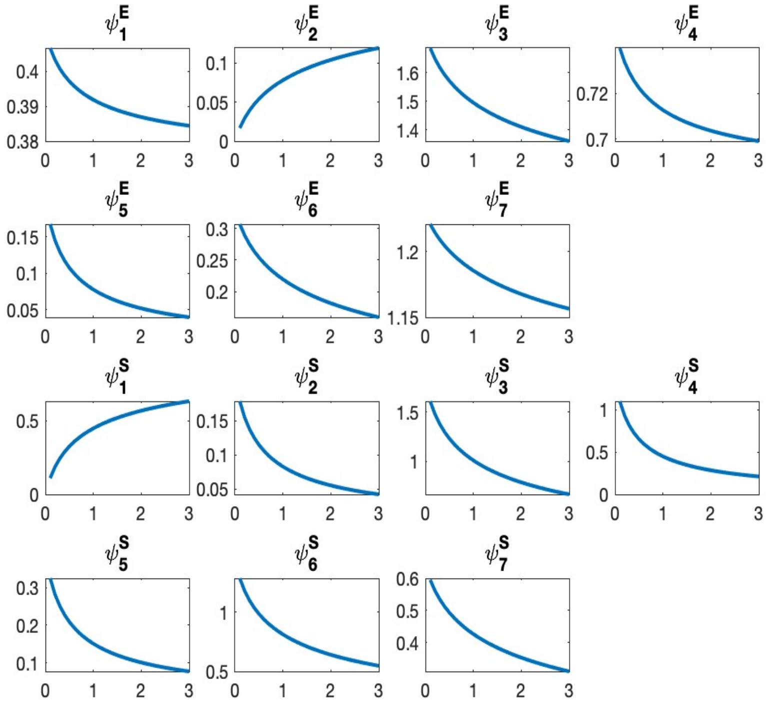
Note: This graph shows when the equipment adjustment cost varies from 0.1 to 3, how do all parameters in policy functions change. We randomize the structure adjustment cost, information friction and equipment/structure correlated factors. However, we do limit the structure adjustment cost and information friction to be positive, and both correlated factors between ± 1.

shocks.

Information friction influences investments through firms' expectations of future productivity. The noisier the signal, the lower the learning rate firms will apply to it, leading them to rely more on current fundamentals for their predictions. Ultimately, the impact of information friction will manifest in investment through ψ_3^E and ψ_3^S. On the other hand, the correlated factors, γ_E and γ_S, will non-trivially influence firm investment in equipment and structures. A positive γ_E (γ_S) indicates higher productivity, higher distortion, and lower investment in equipment (structure), and vice versa. This relationship becomes clearer when these factors are the only frictions. For instance, the equipment-correlated factor can independently generate MRPE dispersion proportional to $\gamma_E^2 \sigma_a^2$. Therefore, the closer the equipment wedge moves in tandem with productivity, the greater the ex-post MRPE dispersion. A similar relationship is observed in the case of structure.

Finally, idiosyncratic and permanent shocks will linearly impact firm investments, with the magnitude of their effects depending on the coefficients in the policy functions. Moreover, there are two forces in our model setting that allow equipment (structure) shocks to enter into the policy function of structure (equipment). The first one is the Cobb-Douglas form between equipment and structure that guarantees the nominal values of these two assets to be in a fixed ratio in production. The second one is the non-zero covariance

Figure D.4: How Do Structure Adjustment Cost Affect Policy Functions?



Note: This graph shows when the structure adjustment cost varies from 0.1 to 3, how do all parameters in policy functions change. We randomize the structure adjustment cost, information friction and equipment/structure correlated factors. However, we do limit the structure adjustment cost and information friction to be positive, and both correlated factors between ± 1.

setting in the i.i.d. and permanent shocks. Firm i is aware that both i.i.d. shocks and permanent shocks are correlated across different types of capital. Consequently, when firm i realizes that it faces an unexpected policy burden in the current period, which results in higher costs for renting a building (structure), it may also anticipate higher expenses when renting equipment.

D.3 Estimating the Whole System

In this section, we provide the algorithm that we used in the baseline estimation. We first write down the equipment (structure) policy function, investment growth, productivity realization and expectation law of motion:

$$\widetilde{e}_{i,t+1} - \psi_3^E(1+\gamma_E)\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) = \psi_1^E\widetilde{e}_{it} + \psi_2^E\widetilde{s}_{it} + \psi_4^E\epsilon_{i,t+1}^E + \psi_5^E\epsilon_{i,t+1}^S + \psi_6^E\chi_i^E + \psi_7^E\chi_i^S \tag{D.33}$$

$$\widetilde{s}_{i,t+1} - \psi_3^S(1+\gamma_S)\mathbb{E}_{it}(\widetilde{a}_{i,t+1}) = \psi_1^S\widetilde{s}_{it} + \psi_2^S\widetilde{e}_{it} + \psi_4^S\epsilon_{i,t+1}^S + \psi_5^S\epsilon_{i,t+1}^E + \psi_6^S\chi_i^S + \psi_7^S\chi_i^E \tag{D.34}$$

$$-\widetilde{e}_{i,t+1} + \iota_{i,t+1}^E = -1\cdot\widetilde{e}_{it} \tag{D.35}$$

$$-\widetilde{s}_{i,t+1} + \iota_{i,t+1}^S = -1\cdot\widetilde{s}_{it} \tag{D.36}$$

$$a_{it+1} = \rho a_{it} + \mu_{it+1} \tag{D.37}$$

$$\mathbb{E}_{it}a_{it+1} = \rho a_{it} + \left(1 - \frac{\mathrm{V}}{\sigma_\mu^2}\right)\mu_{it+1} + \left(1 - \frac{\mathrm{V}}{\sigma_\mu^2}\right)e_{it+1} \tag{D.38}$$

We construct the matrix system using the above equations as:

$$BX_{i,t+1} = CX_{it} + DU_{i,t+1} \tag{D.39}$$

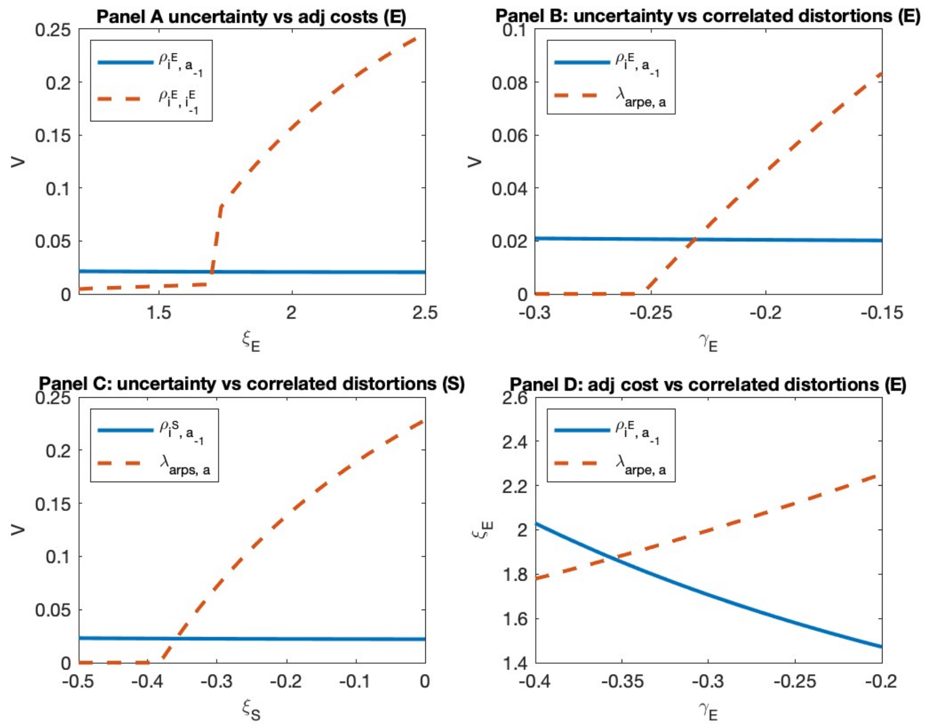
$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\psi_3^E(1+\gamma_E) \\ 0 & 1 & 0 & 0 & 0 & -\psi_3^S(1+\gamma_S) \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad X_{it+1} = \begin{bmatrix} \tilde{e}_{it+1} \\ \tilde{s}_{it+1} \\ \iota_{it+1}^E \\ \iota_{it+1}^S \\ a_{it+1} \\ \mathbb{E}_{it}(a_{it+1}) \end{bmatrix}$$

$$C = \begin{bmatrix} \psi_1^E & \psi_2^E & 0 & 0 & 0 & 0 \\ \psi_2^S & \psi_1^S & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 0 & \rho & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & \psi_4^E & \psi_5^E & \psi_6^E & \psi_7^E \\ 0 & 0 & \psi_5^S & \psi_4^S & \psi_7^S & \psi_6^S \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1-\frac{V}{\sigma_\mu^2} & 1-\frac{V}{\sigma_\mu^2} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad U_{i,t+1} = \begin{bmatrix} \mu_{i,t+1} \\ error_{i,t+1} \\ \varepsilon_{i,t+1}^E \\ \varepsilon_{i,t+1}^S \\ \chi_i^E \\ \chi_i^S \end{bmatrix}$$

D.4 Identification of the Baseline Model

In this section, we discuss the identification for parameters in block two, as identifying block one and three is straight-forward. Given the complexity of our model, we cannot solve the model analytically (even with special cases) and obtain the expressions for the mapping from moments to parameters. Instead, we will implement numerical exercises, using isomoment curves to illustrate the intuition behind the identification. In Figures 11 and 12, we use isomoment curves to demonstrate how parameters can be uniquely identified by these moments. In each graph, there is an orange dashed line and a blue line, each of them representing a moment with a deterministic value. Along these lines, various combinations of two different parameters of interest are shown. We vary the value of the parameter on the horizontal axis and plot the change of another parameter while keeping the value of the moment fixed.

Figure D.5: Isomoment Curves 1: Quantitative Model

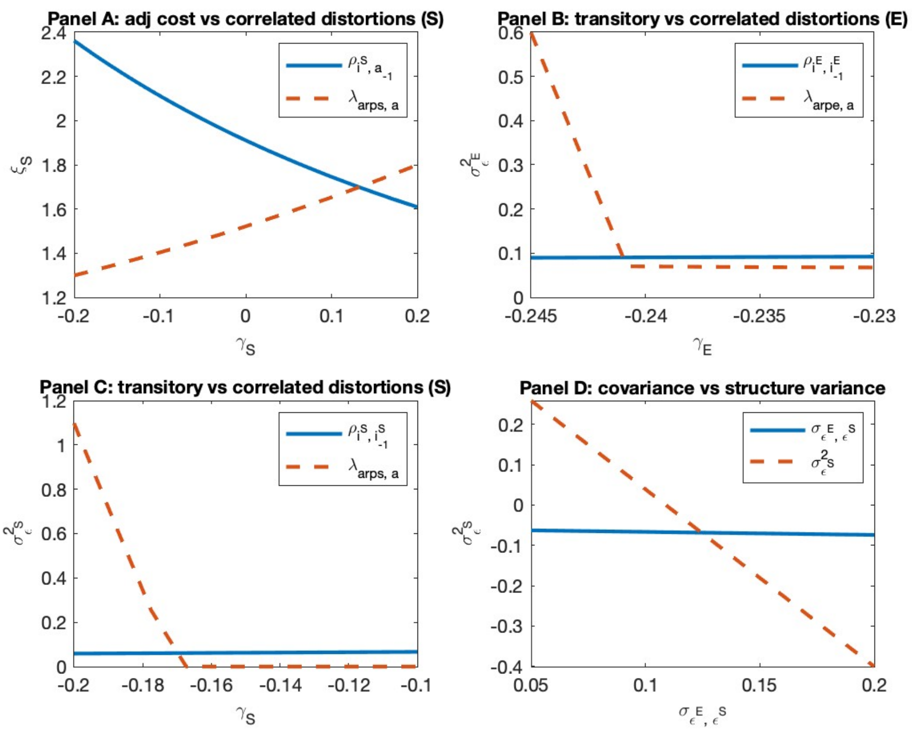


Information Friction and Adjustment Costs In Figure 11, panel A, the orange dashed line represents the auto-correlation of equipment investment growth rate, while the blue line depicts the correlation between equipment investment growth rate and the last productivity. We then vary the adjustment cost of equipment while keeping $\rho_{\Delta e,a_{-1}}$ and $\rho_{\Delta e,\Delta e_{-1}}$ constant and observe how the signal precision V changes. In the figure, the line of $\rho_{\Delta e,\Delta e_{-1}}$ is upward sloping, since the larger the equipment adjustment it is, the greater the $\rho_{\Delta e,\Delta e_{-1}}$ it will be. So in order to keep $\rho_{\Delta e,\Delta e_{-1}}$ constant along with the line, a larger information friction need to kick in to balance out the effect of the adjustment cost. On the other side, $\rho_{\Delta e,a_{-1}}$ seems to be a good moment to identify V, since for different values of equipment adjustment cost, the moment implies almost a constant number of information friction.

Correlated Factors and Information Friction In Figure 11, panel B, we choose $\rho_{\Delta e,a_{-1}}$ and $\rho_{mrpe,a}$ to discuss the identification issue with the equipment correlated factor, γ_E, and information friction, V. The logic for the moment $\rho_{\Delta e,a_{-1}}$ is similar: when γ_E changes, $\rho_{\Delta e,a_{-1}}$ seems to deliver roughly the same value of information friction. However, when γ_E becomes greater, i.e. less negative, firms face less distortions induced by the correlated factor, so they will react more on fundamental shocks. Hence, in order to keep $\rho_{mrpe,a}$ constant, we need a larger information friction to offset the effect.

Adjustment Costs and Correlated Factors In Figure 11, panel D we use the correlation between equipment investment growth and past productivity and correlation between ARPE and current fundamental to pin down the equipment correlated factor and the adjustment cost. For the orange dashed line, as the γ_E increases, the $\lambda_{arpe,a}$ becomes bigger as firms react more on their productivity shocks. Hence, we need a greater adjustment cost ξ_E to keep this moment stable. On the other hand, when γ_E increases, firms rely less on their past fundamentals, so we need a less adjustment cost to fixed this, in a same logic.

Figure D.6: Isomoment Curves 2: Quantitative Model



Correlated Factor and Idiosyncratic Shocks To disentangle correlated and uncorrelated idiosyncratic shocks, γ_E and $\sigma^2_{\varepsilon^E}$, in Figure 12 panel B, we plot the isomoment curves of $\rho_{\Delta e,\Delta e_{-1}}$ and $\rho_{mrpe,a}$. The graph shows that the autocorrelation of investment growth does not depend on the value of the equipment correlated factor. The $\rho_{mrpe,a}$ is downward slopping since larger γ_E will apparently make the $\rho_{mrpe,a}$ greater, so we will need a smaller variance of the shocks to force firms to react more on the current fundamentals. As before, this insures that there is a unique combination of equipment correlated factor and idiosyncratic shock which are consistent with both moments.

Idiosyncratic Matrix Finally, what is left is just how to identify two idiosyncratic shocks and their covariance for equipment and structure. Since different in their share and adjustment costs, the parameters in front of the two iid shocks are different numerically, allowing us to pin down each of them with variance of equipment and structure investment growth.

Finally, after pin down every other parameters in block two, we can simply use the covariance between equipment and structure investment growth to pin down the covariance of iid shocks, $\sigma_{\varepsilon^E \varepsilon^E}$.

D.5 How does capital substitutability affects investment behavior?

We now study how capital substitutability affects firm's investment behavior of different capitals. In particular, we are interested in how the coefficients of policy functions change with the elasticity of capital substitution γ under empirically relevant model parameters and how these changes might affect parameter estimates when we are targeting the fixed set of moments. To do so, we simulate how these coefficients change with γ in an economy where adjustment costs (ξ_E and ξ_S) are positive and correlated distortions are negative with $\gamma_E < \gamma_S$. All the parameters are empirically relevant and corresponds to the estimates in Section 7.

We first rewrite the Euler equations to examine the channels through which various factors affect the investment in each type of capital. Using equipment as an example, the equilibrium investment decisions can be expressed as the sum of four effects:

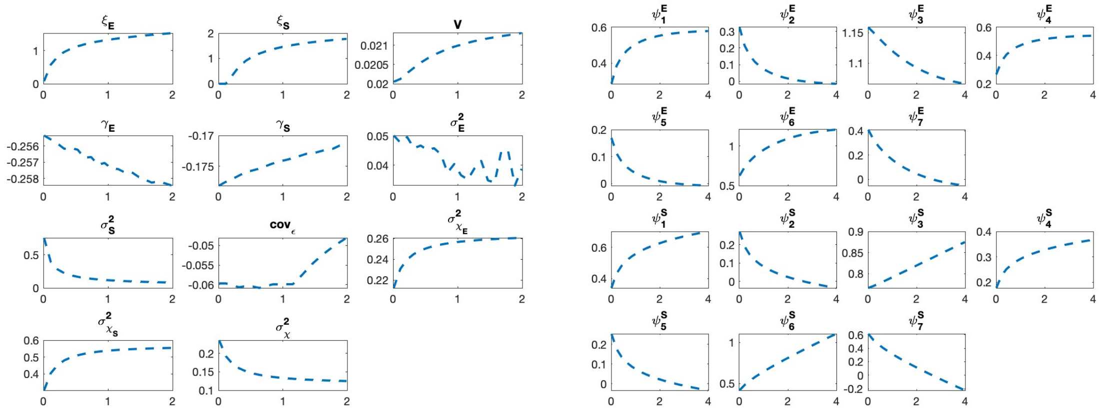
$$\begin{aligned} e_{it} =& \underbrace{\mathbb{E}_{t-1}[p_{it}+y_{it}]}_{\text{size effect}} + \underbrace{\frac{1-\gamma}{\gamma}\Omega_S\left(s_{it}-e_{it}\right)}_{\text{substitution effect}} + \underbrace{\frac{\xi_E}{\overline{\Pi}_1}\left(\mathbb{E}_{t-1}e_{it+1}-e_{it}\right) - \frac{\xi_E}{\beta\overline{\Pi}_1}\left(e_{it}-e_{it-1}\right)}_{\text{adjustment distortion effect}} \\ & - \underbrace{\frac{\left(1-\beta\left(1-\delta_E\right)\right)}{\beta\overline{\Pi}_1}\left(\gamma_E\mathbb{E}_{t-1}[a_{it}]+\varepsilon^E_{it}+\chi^E_i\right)}_{\text{input distortion effect}}. \end{aligned} \tag{D.40}$$

The size effect is intuitive: firms with larger expected sales would invest more in equipment for production. The substitution effect depends on γ: when $\gamma < 1$, equipment and structures are complements, so firms with more structure than equipment (compared to the steady state) would invest more in equipment to complement the production in equilibrium. The converse is true for $\gamma > 1$. The adjustment and input distortion effects act as increases in the effective price of equipment, discouraging investment due to higher marginal adjustment costs and equipment-related distortions.

Adjustment Costs Figure D.7 shows that, all else equal, as γ becomes larger, a firm with already installed equipment will retain more of its installed equipment ($\frac{d\psi_1^E}{d\gamma} > 0$) and have less structure ($\frac{d\psi_2^S}{d\gamma} < 0$) in the next period. The same conclusion holds for structure as well.

The reason for $\frac{d\psi_1^E}{d\gamma} > 0$ is intuitive. As γ becomes larger, all capital inputs become more substitutable in production such that a firm need not adjust much of its installed equipment to achieve a higher output next period. In other words, a firm would face the same marginal adjustment cost but a smaller marginal benefit of adjustment. Therefore, more previously installed equipment would be retained and $\psi_1^E = \frac{\partial e_{it}}{\partial e_{it-1}}$ would be larger. At the same time, as inputs are less complementary in production, it becomes less necessary for the firm to simultaneously own more structure to complement the retained equipment

Figure D.7: How γ Affects Estimation Results



(a) Estimated Parameters (b) Estimated Policy Function

Note: This figure shows the measured allocative efficiency with heterogeneous techniques. Figure 5 provides the time series evidence for AE, while Figure 6 shows how AE changes with different values of the elasticity of capital substitution.

and $\frac{d\psi_2^S}{d\gamma} < 0$. The intuition can be best understood in the two limits: as $\gamma \to 0$, equipment and structure are perfect complements (Leontief) such that same proportions of structure is needed to complement the retained equipment in production, i.e., $\psi_1^E = \psi_2^S$. (2) as $\gamma \to \infty$, equipment adjustment costs have no bearing on the investment in structure due to perfect substitutability.

The own adjustment effect leads to an increase in the auto-correlation of capital stock of each type ($\rho_{e,e-1}$ and $\rho_{s,s-1}$) but a decrease in the auto-correlation of investment of each type ($\rho_{\Delta e,\Delta e-1}$ and $\rho_{\Delta s,\Delta s-1}$). This is intuitive since as capital stock becomes less frequently adjusted, investment behaves more like "white noise". Thus, if we are only targeting $\rho_{\Delta e,\Delta e-1}$ and $\rho_{\Delta s,\Delta s-1}$, a lower calibrated γ will lead to a smaller estimated adjustment costs to rationalize the same auto-correlation in the data. However, notice that a lower $\gamma < 1$ might also leads to an increase in $\sigma^2_{\Delta e}$ and $\sigma^2_{\Delta s}$ as well as the correlatedness of $\sigma^2_{\Delta e,\Delta s}$. Therefore, when targeting all moments, the estimated adjustment cost need not monotonically decrease.

Correlated Distortions from Productivity We now examine how higher substitutability between capital types shapes firms' responses to expected productivity shocks due to correlated distortions. Consider an empirically relevant case where $\gamma_E > \gamma_S > 0$, such that a firm facing an expected productivity shock would experience a greater increase in the effective cost of equipment than that of structures. This intuition is best illustrated in a case without any friction but with correlated distortions, where we can write:

$$s_{it} - e_{it} = \gamma \left[\frac{(1-\beta(1-\delta_E))}{\beta\overline{\Pi}_1}\gamma_E - \frac{(1-\beta(1-\delta_S))}{\beta\overline{\Pi}_1}\gamma_S \right] \mathbb{E}_{t-1}[a_{it}].$$

Since $\delta_E > \delta_S$, we will have $\psi_3^S - \psi_3^E = \frac{d(s_{it}-e_{it})}{d\mathbb{E}_{t-1}[a_{it}]} > 0$ as long as $\gamma_E > \gamma_S$. Additionally, note that in this case, the substitution effect is stronger when elasticity is larger, such that $\frac{d(\psi_3^S-\psi_3^E)}{d\gamma} = \frac{1}{\beta\Pi_1}\left[(1-\beta(1-\delta_E))\gamma_E - (1-\beta(1-\delta_S))\gamma_S\right] > 0$. Therefore, when capital types are more substitutable, a firm would use more structures relative to equipment when facing the same expected productivity shock. As the direct size effect (and the interaction with other channels) of expected productivity on investment quantitatively remains unchanged with respect to a change in γ, the dominating substitution channel results in $\frac{d\psi_3^S}{d\gamma} > 0$ and $\frac{d\psi_3^E}{d\gamma} < 0$.

These movements suggest that as γ becomes smaller, ψ_3^E becomes larger and ψ_3^S becomes smaller. this would increase $\rho_{\Delta e,a-1}$ in the model, but lower $\rho_{\Delta s,a-1}$. we would need to have an increase in γ_E and a decline in γ_S to rationalize the $\rho_{\Delta e,a-1}$ and $\rho_{\Delta s,a-1}$ in the data. Also, a decrease in γ would create a much larger drop in the auto-correlation of productivity and equipment investment than the increase in the the auto-correlation of productivity and structures investment. Therefore, this force could overall lead to a decline in information friction $\mathbb{V}$ in the estimates, which boosts the transmission of an actual productivity shocks and the perceived expectation of it, and thus leads to a higher auto-correlation and rationalize the moment in the data.

Response to Idiosyncratic and Permanent Distortions We now analyze how capital substitutability affects investment responses, specifically how investment in one type of capital responds to distortions affecting its own type and other types of capital. Since a firm's response to ε_{it}^X and ξ_i^X is identical in the linearized policy, we will focus on the behavior of $\psi_4^E \equiv \frac{de_{it}}{d\varepsilon_{it}^E}$ and $\psi_5^E \equiv \frac{de_{it}}{d\varepsilon_{it}^S}$. In an economy without adjustment costs, we can write firm's equipment investment response to ε_{it}^E and ε_{it}^S using Equation 6.13:

$$
\begin{aligned}
\frac{de_{it}}{d\varepsilon_{it}^E} &= \alpha\left[\Omega_E\frac{de_{it}}{d\varepsilon_{it}^E} + \Omega_S\frac{ds_{it}}{d\varepsilon_{it}^E}\right] + \frac{1-\gamma}{\gamma}\Omega_S\left(\frac{ds_{it}}{d\varepsilon_{it}^E} - \frac{de_{it}}{d\varepsilon_{it}^E}\right) - 1 \\
&= \underbrace{-\frac{\alpha\Omega_E}{1-\alpha}}_{\text{size effect}} \underbrace{+(1-\gamma)\Omega_S}_{\text{substitution effect}} \underbrace{-1}_{\substack{\text{input}\\\text{distortion}\\\text{effect}}}, \\
\frac{de_{it}}{d\varepsilon_{it}^S} &= \alpha\left[\Omega_E\frac{de_{it}}{d\varepsilon_{it}^S} + \Omega_S\frac{ds_{it}}{d\varepsilon_{it}^S}\right] + \frac{1-\gamma}{\gamma}\Omega_S\left(\frac{ds_{it}}{d\varepsilon_{it}^S} - \frac{de_{it}}{d\varepsilon_{it}^S}\right) \\
&= \underbrace{-\frac{\alpha\Omega_S}{1-\alpha}}_{\text{size effect}} \underbrace{-(1-\gamma)\Omega_S}_{\text{substitution effect}}.
\end{aligned}
$$

Notice that the input distortion on equipment acts like an increase in its effective price, leading to a proportional reduction in equipment demand. Moreover, distortions on any type of capital increase the firm's marginal costs and reduce the firm's size, thereby discouraging equipment investment. Furthermore, γ governs the substitution effect. When $\gamma < 1$, the firm will reduce equipment investment less in response to ε_{it}^E but might reduce it more in response to ε_{it}^S due to the complementarity in production. And the opposite is true for the substitute case. Formally,

$$\frac{\partial \psi_4^E}{\partial \gamma} = \frac{\partial \left(\frac{\partial e_{it}}{\partial \varepsilon_{it}^E}\right)}{\partial \gamma} = -\Omega_S < 0, \text{ and } \quad \frac{\partial \psi_5^E}{\partial \gamma} = \frac{\partial \left(\frac{\partial e_{it}}{\partial \varepsilon_{it}^S}\right)}{\partial \gamma} = \Omega_S > 0,$$

which shows that as γ increases, the substitution effect causes a reduction in the investment response of an asset to distortions in the same asset but an increase in the response to distortions in other assets.

As γ decreases, the own effect increases (but remains < 0), while the cross effect decreases (and may or may not be < 0). To match the variance and covariance of the data moments, we need more volatile distortions in the cross-section and a stronger negative correlation between distortion types.

E Supplementary Results, Tables and Figures

E.1 SMM Performance

We show our SMM performance in the following table. The Data column shows the moment values we are targeting. The right column with blue color denotes the moment values our model generate. As we can see, our model matches the data quite well. In general, the model matches better in the U.S. than in India. Except for the variance covariance matrix of investment growth, which aligns closer to the model when γ is larger. [46]

In the Tables below, we also show the contribution of each individual channel in US when moments are changed with different values of γ. From the results we can see that, regardless of these details of specifications, we can always draw a conclusion that adjustment costs and information friction can not account for the more misalloaction measured by smaller number of elasticity.

F Baseline Model with Heterogeneous Financial Friction

F.1 Model Set-up and Results

Chaney et al. (2012) documented that a larger share of real estate in a firm's capital leads to an increase in their capital expenditure. This indicates that structures might be more commonly used as collateral compared to equipment. Hence, if collateral constraints act as costs of production, this empirical pattern could imply that structures are less misallocated than equipment since there is less liquidity cost associated with holding structures. Yet, it

[46] Note that in the baseline results, we show estimations using the same moments. However, similar to our static measurement framework, different values of γ alter productivity estimates and, subsequently, the moments. There are two reasons for using fixed moments. First, following the arguments from D. R. Baqaee and Farhi (2020) and Hulten (1978), productivity changes are independent of elasticity in the first order, and we solve our model using a first-order approximation. Second, fixing the moments helps us analyze the mechanism. We provide estimation results as robustness checks.

Table E.5: SMM Estimation Performance

Description	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 4$	Data
$\rho_{\iota^e,\iota^e_{-1}}$	-0.3373	-0.3372	-0.3372	-0.3426
$\rho_{\iota^s,\iota^s_{-1}}$	-0.3382	-0.3364	-0.3364	-0.3302
σ^2_e	0.0696	0.0687	0.0687	0.0465
σ^2_s	0.0756	0.0790	0.0790	0.1003
$\sigma^2_{e,s}$	0.0225	0.0181	0.0181	0.0175
$\rho_{\iota^e,\iota^e_{-1}}$	0.5084	0.5090	0.5090	0.5076
$\rho_{\iota^s,\iota^s_{-1}}$	0.3061	0.3060	0.3060	0.3080
$\rho_{arpe,a}$	0.0961	0.1072	0.1072	0.0914
$\rho_{arps,a}$	0.1036	0.0936	0.0936	0.1082
σ^2_{arpe}	0.4193	0.4192	0.4193	0.4193
σ^2_{arps}	0.7524	0.7525	0.7524	0.7524
$\sigma^2_{arpe,arps}$	0.2437	0.2437	0.2437	0.2437

Note: This table shows the matching accuracy between model and data from the SMM estimation. The left panel describes its performance in the U.S., and the right panel is India.

could also be that firms internalize this empirical pattern and hold too many structures and too few pieces of equipment, making equipment less misallocated compared to structures.

In this section, we extend our baseline model to incorporate the heterogeneous collateral constraints of structures and equipment. For keeping our model tractable, we model financial frictions as a continuous operation cost and firms need to hold a certain amount of the liquidity asset with a relatively low exogenous return rate ($R < \frac{1}{\beta}$). Specifically, the liquidity cost is given below

$$\Upsilon(E_{it+1}, S_{it+1}, B_{it+1}) = \hat{\nu} E_{it+1}^{\omega_e} S_{it+1}^{\omega_s} B_{it+1}^{\omega_b} \tag{F.1}$$

where $\hat{\nu}$ is the unit-cost of liquidity cost. ω_e and ω_s determine how financial costly to use equipment and structures in firm's production. Intuitively, both of them are positive numbers. Similarly, $\omega_b > 0$ (< 0) indicates potential cost (benefit) of holding the liquidity asset. After optimizing the choice of B_{it+1}, the liquidity cost can be written as:

$$\Upsilon(E_{it+1}, S_{it+1}) = \nu E_{i,t+1}^{\frac{\omega_e}{1-\omega_b}} S_{i,t+1}^{\frac{\omega_s}{1-\omega_b}} \tag{F.2}$$

in which ν is increasing in $\hat{\nu}$. If $\frac{\omega_e}{1-\omega_b} > 1$ ($\frac{\omega_s}{1-\omega_b} > 1$), then the marginal liquidity cost is increasing with larger equipment (structures) stocks, and vise versa. The detail of model with heterogeneous collateral constraints are attached in Appendix I.

We need to use firm-level liquidity holdings data together with production-side data to estimate the heterogeneous collateral constraints David and Venkateswaran (2019). We use the covariance of firm's debt holding and equipment (structures) stocks to pin down the

Table E.6: Contributions of Frictions on arpe (arps) Dispersions with $\gamma = 0.3$ (US)

	ξ_E	ξ_S	V	γ_E	γ_S	$\sigma^2_{\varepsilon^E}$	$\sigma^2_{\varepsilon^S}$	$\sigma_{\varepsilon^E,\varepsilon^S}$	$\sigma^2_{\chi^E}$	$\sigma^2_{\chi^S}$	σ_{χ^E,χ^S}	Total
E	0.05		0.07		0.11		0.02	0.03	1.09			1.28
	4.08		5.76		8.40		1.39	1.99	85.47			100.00
S		0.13				0.32	0.04	0.03		3.47		3.64
		3.68		0.00		8.84	1.23	0.70		95.25		100.00
Cov							−0.04	0.03			−1.36	−1.34
							2.69	−1.90			101.38	100.00
TFP	0.01	0.01	0.01		0.02	0.02	−0.00	0.01	0.23	0.19	−0.23	0.26
	3.63	3.98	5.07	0.00	7.40	9.55	−0.03	4.33	90.90	74.04	−89.05	100.00

Table E.7: Contributions of Frictions on arpe (arps) Dispersions with $\gamma = 4$ (US)

	ξ_E	ξ_S	γ_E	γ_S	$\sigma^2_{\varepsilon^E}$	$\sigma^2_{\varepsilon^S}$	cov_ε	V	$\sigma^2_{\chi^E}$	$\sigma^2_{\chi^S}$	cov_χ	varape
E	0.05		0.02		0.12		−0.00	0.03	0.26			0.38
	13.60		6.23		31.32		−0.04	6.59	68.35			100.00
S		0.03		0.03		0.05	−0.00	0.03		0.28		0.39
		6.55		7.21		13.44	−0.11	6.49		72.66		100.00
Cov							−0.00	0.03			0.25	0.36
							−0.34	6.92			69.62	100.00
TFP	0.02	0.00	0.01	0.00	0.05	0.01	−0.00	0.01	0.09	0.06	−0.03	0.17
	11.84	1.70	5.50	1.88	27.62	3.51	−0.01	6.51	51.88	33.73	−15.12	100.00

two numbers in power in the liquidity cost. Specifically, we run the following regression:

$$\log(B_{it}) = \beta_0 + \beta_1 \log(E_{it}) + \beta_2 \log(S_{it}) + \varepsilon_{it} \tag{F.3}$$

where B_{it} is firm's debt holding, and E_{it} and S_{it} are equipment and structures stock. This empirical specification is different from Gopinath et al. (2017) whose dependent variable is the leverage ratio, i.e. B/K, since our stylized financial constraints do not include the size-dependent factor. We also try to run this regression after taking first difference on all variables. The results can be found in the Table F.10 below.

From the table we can see that the estimated $\hat{\beta}_1$ is always smaller than $\hat{\beta}_2$. This indicates that in average for a firm, its debt holding is more correlated with its structures than equipment stock. This pattern is consistent when variables are first differenced. Hence, $\hat{\beta}_1$ and $\hat{\beta}_2$ will be used as two moments to discipline our model with heterogeneous collateral constraints. We also match our model with $\sigma^2_{\Delta mrpe}$ and $\sigma^2_{\Delta mrps}$, in order to pin down all

Table E.8: Contributions of Frictions on arpe (arps) Dispersions with $\gamma = 0.3$ (US) with Corresponding Moments

	ξ_E	ξ_S	γ_E	γ_S	$\sigma^2_{\varepsilon^E}$	$\sigma^2_{\varepsilon^S}$	cov_ε	V	$\sigma^2_{\chi^E}$	$\sigma^2_{\chi^S}$	cov_χ	varape
E	0.02		0.01		0.01		0.00	0.03	1.05			1.10
	1.58		1.09		0.73		0.22	2.30	95.40			100.00
S		0.43		0.00		1.35	0.01	0.03		2.60		2.98
		14.43		0.09		45.42	0.42	0.85		87.30		100.00
Cov							-0.01	0.03			-1.23	-1.15
							0.64	-2.20			106.90	100.00
TFP	0.00	0.03	0.00	0.00	0.00	0.10	0.00	0.01	0.23	0.14	-0.21	0.21
	1.46	15.73	1.01	0.10	0.68	48.72	0.01	5.23	106.33	67.40	-97.88	100.00

Table E.9: Contributions of Frictions on arpe (arps) Dispersions with $\gamma = 4$ (US) with Corresponding Moments

	ξ_E	ξ_S	γ_E	γ_S	$\sigma^2_{\varepsilon^E}$	$\sigma^2_{\varepsilon^S}$	cov_ε	V	$\sigma^2_{\chi^E}$	$\sigma^2_{\chi^S}$	cov_χ	varape
E	0.07		0.01		0.23		0.00	0.02	0.20			0.31
	21.38		1.86		75.67		-0.04	5.73	65.57			100.00
S		0.02		0.01		0.06	-0.00	0.02		0.24		0.32
		7.35		4.30		18.54	-0.09	5.59		74.67		100.00
Cov							-0.00	0.02			0.21	0.29
							-0.29	6.01			69.76	100.00
TFP	0.03	0.00	0.00	0.00	0.09	0.01	-0.00	0.01	0.07	0.05	-0.02	0.14
	18.30	1.92	1.63	1.13	66.50	4.88	-0.01	5.65	49.59	34.90	-15.14	100.00

four parameters: ω_e, ω_s, ω_b and $\hat{\nu}$. The estimation results are attached in Table F.11.

From the result, $\omega_e > \omega_b$ holding the same units of equipment will be more costly compared to structures for firms. This result is consistent with the empirical results found in Chaney et al. (2012): when the ratio of real estate is higher in firm's capital stock, firm's investment will be higher as well. Eventually, both empirical and estimation results review the fact that equipment faces more tighten collateral constraints than structures, so this collateral constraints heterogeneity is less likely to be able to explain why structures are more misallocated than equipment.

Table F.10: Firm's Debt Holdings and Structures/Equipment Stocks

	(1)	(2)	(3)
$\log(Structures)$	0.308***	0.359***	
	(0.007)	(0.009)	
$\log(Equipment)$	0.604***	0.586***	
	(0.007)	(0.012)	
$\Delta\log(Structures)$			0.357***
			(0.012)
$\Delta\log(Equipment)$			0.556***
			(0.018)
Year-Sector Fixed Effect	Yes	Yes	Yes
Firm Fixed Effect	No	Yes	Yes
Observations	55,111	53,928	43,253
R^2	0.762	0.907	0.335

F.2 Solving the Model

We first define the liquidity costs as $\Upsilon(E_{i,t+1}, S_{i,t+1}, B_{i,t+1})$ which is given by:

$$\Upsilon\left(E_{it+1}, S_{it+1}, B_{it+1}\right) = \hat{\nu} E_{i,t+1}^{\omega_e} S_{i,t+1}^{\omega_s} B_{i,t+1}^{\omega_b}$$

Then, the firm i's profit maximization problem is given by:

$$\begin{aligned}\mathcal{V}\left(E_{it}, S_{it}, B_{it}, \mathcal{I}_{it}\right) = \max_{E_{i,t+1}, S_{i,t+1}, B_{i,t+1}} \mathbb{E}_{it} \Bigg[& G A_{it}\left(E_{it}^{\alpha_E} S_{it}^{\alpha_S}\right)^{\alpha} + R B_{it} \\ & - T_{i,t+1}^{E} E_{i,t+1}\left(1-\beta\left(1-\delta_E\right)\right) - \Phi\left(E_{i,t+1}, E_{it}\right) \\ & - T_{i,t+1}^{S} S_{i,t+1}\left(1-\beta\left(1-\delta_S\right)\right) - \Phi\left(S_{i,t+1}, S_{it}\right) \Bigg] \\ & - B_{i,t+1} - \Upsilon\left(E_{it+1}, S_{it+1}, B_{it+1}\right) + \beta \mathbb{E}_{it}\left[\mathcal{V}\left(E_{i,t+1}, S_{i,t+1}, B_{i,t+1}, \mathcal{I}_{it+1}\right)\right]\end{aligned}$$

We start to solve this problem by pinning down the optimal decisions for bonds borrowing or lending (depending if B_{it} is positive or negative). The first order condition for $B_{i,t+1}$ can be expressed as:

$$\begin{aligned} \left[B_{i,t+1}\right] : 1 - \hat{\nu} E_{i,t+1}^{\omega_e} S_{i,t+1}^{\omega_s} \cdot \omega_b B_{i,t+1}^{\omega_b - 1} &= \beta R \\ \Rightarrow B_{i,t+1} &= \left[\frac{1-\beta R}{\hat{\nu}\omega_b \cdot E_{i,t+1}^{\omega_e} S_{i,t+1}^{\omega_s}}\right]^{\frac{1}{\omega_b - 1}} \\ \Rightarrow B_{i,t+1}^{\omega_b} &= \left[\frac{1-\beta R}{\hat{\nu}\omega_b \cdot E_{i,t+1}^{\omega_e} S_{i,t+1}^{\omega_s}}\right]^{\frac{\omega_b}{\omega_b - 1}} \end{aligned}$$

Table F.11: Financial Constraints Estimation Results

Parameters	Description	U.S.
ρ	Persistence of productivity	0.94
σ_μ^2	Shock to productivity	0.04
ξ_E	Equipment adjustment cost	2.94
ξ_S	Structure adjustment cost	0.63
$\mathbb{V}$	Signal precision	0.03
γ_E	Equipment correlated factor	-0.21
γ_S	Structure correlated factor	-0.29
$\sigma_{\varepsilon_E}^2$	Equipment transitory	0.13
$\sigma_{\varepsilon_S}^2$	Structure transitory	0.01
$cov(\varepsilon_E, \varepsilon_S)$	i.i.d. correlation factor	-0.03
$\sigma_{\chi_E}^2$	Equipment permanent factor	0.24
$\sigma_{\chi_S}^2$	Structure permanent factor	0.57
$cov(\chi_E, \chi_S)$	correlation factor	0.12
ω_e	equipment liquidity cost	0.16
ω_s	structures liquidity cost	0.10
$\hat{\nu}$	liquidity cost unit-cost	0.48
ω_b	debt liquidity cost	0.73

Note: The table reports the results of SMM estimation. The number is intentionally kept as two digits. The column reports the results for the U.S.

Solving the model, the two Euler equations are given by

$$[E_{i,t+1}] : \mathbb{E}_{it}\Bigg[\left(\widetilde{a}_{i,t+1} + (\alpha_E\alpha - 1)\,\widetilde{e}_{i,t+1} + \alpha_S\alpha\widetilde{s}_{i,t+1}\right) + \beta\xi_E\left(\widetilde{e}_{it+2} - \widetilde{e}_{it+1}\right)$$
$$+ \widetilde{\tau}^e_{i,t+1} - \xi_E\left(\widetilde{e}_{i,t+1} - \widetilde{e}_{it}\right) + \Xi_e\widetilde{e}_{i,t+1} + \Xi_s\widetilde{s}_{i,t+1}\Bigg] = 0 \tag{F.4}$$

$$[S_{i,t+1}] : \mathbb{E}_{it}\Bigg[\left(\widetilde{a}_{i,t+1} + (\alpha_S\alpha - 1)\,\widetilde{s}_{i,t+1} + \alpha_E\alpha\widetilde{e}_{i,t+1}\right) - \beta\xi_S\left(\widetilde{s}_{it+2} - \tilde{s}_{it+1}\right)$$
$$- \widetilde{\tau}^s_{i,t+1} - \xi_S\left(\widetilde{s}_{i,t+1} - \tilde{s}_{it}\right) + \Omega_e\widetilde{e}_{i,t+1} + \Omega_s\widetilde{s}_{i,t+1}\Bigg] = 0 \tag{F.5}$$

where Ξ_e, Ξ_s, Ω_e and Ω_s are given by:

$$\Xi_e = -\frac{\frac{\nu\omega_e}{1-\omega_b}E^{\frac{\omega_e}{1-\omega_b}-1}S^{\frac{\omega_s}{1-\omega_b}}}{\beta G\alpha\alpha_E E^{\alpha_E\alpha-1}S^{\alpha_S\alpha}}\cdot\left(\frac{\omega_e}{1-\omega_b}-1\right) \tag{F.6}$$

$$\Xi_s = -\frac{\frac{\nu\omega_e}{1-\omega_b}E^{\frac{\omega_e}{1-\omega_b}-1}S^{\frac{\omega_s}{1-\omega_b}}}{\beta G\alpha\alpha_E E^{\alpha_E\alpha-1}S^{\alpha_S\alpha}}\cdot\left(\frac{\omega_s}{1-\omega_b}\right) \tag{F.7}$$

$$\Omega_e = -\frac{\frac{\nu\omega_s}{1-\omega_b}E^{\frac{\omega_e}{1-\omega_b}}S^{\frac{\omega_s}{1-\omega_b}-1}}{\beta G\alpha\alpha_S E^{\alpha_E\alpha}S^{\alpha_S\alpha-1}}\cdot\left(\frac{\omega_e}{1-\omega_b}\right) \tag{F.8}$$

$$\Omega_s = -\frac{\frac{\nu\omega_s}{1-\omega_b}E^{\frac{\omega_e}{1-\omega_b}}S^{\frac{\omega_s}{1-\omega_b}-1}}{\beta G\alpha\alpha_S E^{\alpha_E\alpha}S^{\alpha_S\alpha-1}}\cdot\left(\frac{\omega_s}{1-\omega_b}-1\right) \tag{F.9}$$

G Heterogeneous Markups and Technologies

G.1 Motivation and Results

In our baseline model, we assume a constant elasticity of substitution across firm outputs, as well as homogeneous factor shares in firm production functions. Here, we extend the model to account for firms having different markups and firm-specific factor shares in production functions. Specifically, firm i's cost minimization problem is given by:

$$\min_{E_{it},S_{it},N_{it},M_{it}} R^E_t T^E_{it} E_{it} + R^S_t T^S_{it} S_{it} + W_t T^N_{it} N_{it} + P^M_{it} M_{it}$$
$$\text{s.t. } Y_{it} \leq A_{it} E_{it}^{\hat{\alpha}^E_{it}\hat{\alpha}\hat{\zeta}} S_{it}^{(1-\hat{\alpha}^E_{it})\hat{\alpha}\hat{\zeta}} N_{it}^{(1-\hat{\alpha})\hat{\zeta}} M_{it}^{1-\hat{\zeta}} \tag{G.1}$$

where $\hat{\alpha}^E_{it}$ and $\hat{\alpha}^S_{it}$ are the equipment and structure share in firm i's capital bundles. M_{it}, P^M_{it} and $\hat{\xi}$ refer to its intermediate input, price and its elasticity. The optimality condition with respect to the intermediate input M_{it} simply yields:

$$P^M_{it} = MC_{it}(1-\hat{\zeta})\frac{Y_{it}}{M_{it}} \Rightarrow \frac{P^M_{it} M_{it}}{P_{it} Y_{it}} = (1-\hat{\zeta})\frac{MC_{it}}{P_{it}} \tag{G.2}$$

which allows us to back out the cross-sectional dispersion in markups using data on intermediate input share of firm's sales. We follow De Loecker, Eeckhout, and Unger (2020) to measure the markups using US Compustat and India ASI datasets. We show the detail of our measurement and math details of firm's cost minimization problem in Appendix G.

The results show that in the US, the markup dispersion is around 0.07 in our sample. This indicates that the unobserved markup in our baseline model can explain around 19.4% of MRPE dispesion and 10.7%, which is a sizable effect. However, the heterogeneous markup does not explain the difference between MRPE and MRPS dispersion. To see this, the first order conditions (in logarithm) for equipment and structures are given by

$$[E_{it}]: \quad \log\frac{P_{it}Y_{it}}{E_{it}} = \log\frac{P_{it}}{MC_{it}} - \log\hat{\alpha}^E_{it} + \tau^E_{it} + \text{ Constant} \tag{G.3}$$
$$[S_{it}]: \quad \log\frac{P_{it}Y_{it}}{S_{it}} = \log\frac{P_{it}}{MC_{it}} - \log(1-\hat{\alpha}^E_{it}) + \tau^S_{it} + \text{ Constant} \tag{G.4}$$

in which the markup dispersion contributes identically to MRPE and MRPS dispersion.

G.2 Solving Markups

Now we extend firm's production function to contain intermediate goods. Even though we assume the equipment and structure share sum up to be one in our baseline model, we relax this assumption here. When the production function is CES, firm's problem can be rewritten as

$$\begin{aligned} \min_{E_{it},S_{it},N_{it},M_{it}} \quad & R_t^E T_{it}^E E_{it} + R_t^S T_{it}^S S_{it} + W_t T_{it}^N N_{it} + P_{it}^M M_{it} \\ \text{s.t. } Y_{it} \leq A_{it} & \left[\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_{Sit}^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\hat{\alpha}\hat{\zeta}} N_{it}^{(1-\hat{\alpha})\hat{\zeta}} M_{it}^{1-\hat{\zeta}} \end{aligned} \tag{G.5}$$

Write down the Lagrangian

$$\begin{aligned} \mathcal{L} = & R_t^E T_{it}^E E_{it} + R_t^S T_{it}^S S_{it} + W_t T_{it}^N N_{it} + P_{it}^M M_{it} \\ & - \lambda_{it} \cdot \left(A_{it} \left[\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_{Sit}^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\hat{\alpha}\hat{\zeta}} N_{it}^{(1-\hat{\alpha})\hat{\zeta}} M_{it}^{1-\hat{\zeta}} - Y_{it} \right) \end{aligned} \tag{G.6}$$

where we temporarily only allow for factor shares heterogeneity between equipment and structures. Their first order conditions can be written down as

$$[E_{it}] \quad R_t^E T_{it}^E = \lambda_{it} \cdot \hat{\alpha}\hat{\zeta} \cdot \frac{Y_{it}}{E_{it}} \cdot \frac{\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}}}{\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_{Sit}^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}}} \tag{G.7}$$

$$[S_{it}] \quad R_t^S T_{it}^S = \lambda_{it} \cdot \hat{\alpha}\hat{\zeta} \cdot \frac{Y_{it}}{S_{it}} \cdot \frac{\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}}}{\alpha_{Eit}^{\frac{1}{\gamma}} E_{it}^{\frac{\gamma-1}{\gamma}} + \alpha_{Sit}^{\frac{1}{\gamma}} S_{it}^{\frac{\gamma-1}{\gamma}}} \tag{G.8}$$

$$[N_{it}] \quad W_t T_{it}^N = \lambda_{it} \cdot (1-\hat{\alpha})\hat{\zeta} \cdot \frac{Y_{it}}{N_{it}} \tag{G.9}$$

$$[M_{it}] \quad P_{it}^M = \lambda_{it} \cdot (1-\hat{\zeta}) \cdot \frac{Y_{it}}{M_{it}} \tag{G.10}$$

The Lagrangian multiplier λ_{it} is in fact the marginal cost, and rearrange the FOCs for equipment and structures

$$[E_{it}] \quad R_t^E T_{it}^E = \frac{MC_{it}}{P_{it}} \cdot \hat{\alpha}\hat{\zeta} \cdot \frac{P_{it} Y_{it}}{E_{it}} \cdot \frac{1}{1 + \left(\frac{\alpha_{Sit}}{\alpha_{Eit}}\right)^{\frac{1}{\gamma}} \left(\frac{S_{it}}{E_{it}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{G.11}$$

$$[S_{it}] \quad R_t^S T_{it}^S = \frac{MC_{it}}{P_{it}} \cdot \hat{\alpha}\hat{\zeta} \cdot \frac{P_{it} Y_{it}}{S_{it}} \cdot \frac{1}{1 + \left(\frac{\alpha_{Eit}}{\alpha_{Sit}}\right)^{\frac{1}{\gamma}} \left(\frac{E_{it}}{S_{it}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{G.12}$$

Taking log on both sides of these two equations and applying the fact that structures and equipment shares add up to one

$$[E_{it}] \quad \log(R_t^E) + \tau_{it}^E = \log(\frac{MC_{it}}{P_{it}}) + \log(\hat{\alpha}\hat{\zeta}) + arpe_{it} - \log\left(1 + \left(\frac{1-\alpha_{Eit}}{\alpha_{Eit}}\right)^{\frac{1}{\gamma}} \left(\frac{S_{it}}{E_{it}}\right)^{\frac{\gamma-1}{\gamma}}\right) \tag{G.13}$$

$$[S_{it}] \quad \log(R_t^S) + \tau_{it}^S = \log(\frac{MC_{it}}{P_{it}}) + \log(\hat{\alpha}\hat{\zeta}) + arps_{it} - \log\left(1 + \left(\frac{1-\alpha_{Sit}}{\alpha_{Sit}}\right)^{\frac{1}{\gamma}} \left(\frac{E_{it}}{S_{it}}\right)^{\frac{\gamma-1}{\gamma}}\right) \tag{G.14}$$

Notice that compared to the Cobb-Douglas case, we still need to deal with the ratio of structures and equipment input. Denoting the share as $\kappa_{it} = \frac{S_{it}}{E_{it}}$ and rearrange and talk log linearization

$$\begin{aligned}
[E_{it}] \quad arpe_{it} - \log(\frac{P_{it}}{MC_{it}}) &= \tau_{it}^E + \log\left(1 + \left(\frac{1-\alpha_{Eit}}{\alpha_{Eit}}\right)^{\frac{1}{\gamma}} \kappa_{it}^{\frac{\gamma-1}{\gamma}}\right) + \text{constant} \\
&\approx \widetilde{\tau}^E + \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{\alpha_E}\right)^{\frac{1}{\gamma}-1} \frac{1}{\alpha_E}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}} \left(\frac{1}{\alpha_E} - 1\right)^{\frac{1}{\gamma}}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\kappa}_{it} + \text{constant}
\end{aligned} \tag{G.15}$$

$$\begin{aligned}
[S_{it}] \quad arps_{it} - \log(\frac{P_{it}}{MC_{it}}) &= \tau_{it}^S + \log\left(1 + \left(\frac{1-\alpha_{Eit}}{\alpha_{Eit}}\right)^{-\frac{1}{\gamma}} \kappa_{it}^{\frac{\gamma-1}{\gamma}}\right) + \text{constant} \\
&\approx \widetilde{\tau}^S + \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{\alpha_E} - 1\right)^{\frac{1}{\gamma}-1} \frac{1}{\alpha_E}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{-\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}} \left(\frac{1}{\alpha_E} - 1\right)^{-\frac{1}{\gamma}}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\kappa}_{it} + \text{constant}
\end{aligned} \tag{G.16}$$

where κ and α_E are corresponding steady states. With pre-determined parameters and γ, we also need to compute $\widetilde{\kappa}_{it}$ from the data. Rest of the logic will be very similar to the Cobb-Douglas case. Specifically, both

$$\widetilde{arpe}_{it} = arpe_{it} - \log(\frac{P_{it}}{MC_{it}}) - \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}} \left(\frac{1}{\alpha_E} - 1\right)^{\frac{1}{\gamma}}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\kappa}_{it} \tag{G.17}$$

$$\widetilde{arps}_{it} = arps_{it} - \log(\frac{P_{it}}{MC_{it}}) - \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}} \left(\frac{1}{\alpha_E} - 1\right)^{-\frac{1}{\gamma}}}{1 + \left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma-1}{\gamma}}} \widetilde{\kappa}_{it} \tag{G.18}$$

need to be inferred from the dataset.

Alternative Method 1 Following Hsieh and Klenow (2009), we can implement an extra assumption, $\tau_{it}^{E} = \tau_{it}^{N}$ or $\tau_{it}^{S} = \tau_{it}^{N}$, which implies

$$arpe_{it} - arpn_{it} = \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}}\left(1-\frac{1}{\alpha_E}\right)^{\frac{1}{\gamma}-1}\frac{1}{\alpha_E}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{\frac{1}{\gamma}}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\kappa}_{it} + \text{constant} \tag{G.19}$$

or

$$arps_{it} - arpn_{it} = \frac{\frac{1}{\gamma}\kappa^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{\frac{1}{\gamma}-1}\frac{1}{\alpha_E}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{-\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\alpha}_{Eit} + \frac{(1-\frac{1}{\gamma})\kappa^{1-\frac{1}{\gamma}}\left(\frac{1}{\alpha_E}-1\right)^{-\frac{1}{\gamma}}}{1+\left(\frac{1-\alpha_E}{\alpha_E}\right)^{\frac{1}{\gamma}}\kappa^{\frac{\gamma-1}{\gamma}}}\widetilde{\kappa}_{it} + \text{constant} \tag{G.20}$$

and we can compute using the data.

Alternative Method 2 We can also assume a equipment specific productivity component, and estimate the Hisk-neutral and equipment specific productivity jointly from the dataset. Then, we re-estimate the model to evaluate the contribution of it.